Instanton in Euclidean non-abelian field of point-like source

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(Anti-)instanton behaviour in Euclidean non-abelian field of the point-like source is studied by analyzing the possible (anti-)instanton deformations as resulted from the variations of its characteristic parameters. The variational principle for searching such crumpled topological configurations is formulated and the problem is resolved then algebraically by the Ritz method (multipole expansion of the deformation fields). The region of (anti-)instanton parameters relevant to the instanton liquid model is investigated in detail. The approximate efficacious method to include such configurations contributing to the appropriate partition function is proposed and verified. Evaluating average energy of Euclidean non-abelian point-like source in the instanton environment (liquid) is performed within the superposition ansatz for the corresponding solutions of the Yang-Mills equations. For the colour singlet dipole this energy escalates linearly with the separation increasing and its tension coefficient develops the magnitude commensurate with that as inferred from lattice QCD.

Introduction

Considerable interest in classical theory of non-abelian gauge fields has been provoked by the discovery of instantons when it becomes clear that conceptually low energy QCD shares many common features with condensed matter physics. These pseudo-particle (PP) solutions of Euclidean classical equations of the Yang-Mills theory, having been properly included in the Euclidean functional integral, turned out very suggestive in illuminating the nature of QCD vacuum and non-perturbative dynamics [1] (the CDG scenario). In particular, it has been realized the QCD ground state is a sophisticated strongly interacting system filled with the condensates of quark-antiquark pairs and gluons. Moreover, experimentally observed particles respect only a part of the full symmetry of QCD. Then similarly to the condensed matter physics we have chiefly two theoretical approaches to explore these systems. One is less theoretical and based on the numerical simulations. Another relies on the construction of effective theories for the low energy degrees of freedom governed by studying the patterns of breaking of underlying symmetries. The most advanced example is given by the chiral symmetry of QCD in the limit of massless quarks. This symmetry is spontaneously broken by the condensation of quarks and anti-quarks in the QCD vacuum. Such a phenomenon (together with many consequences) is explained to a large extent if one surmises the classical solutions of Yang-Mills equations with nontrivial topology ((anti-)instantons with perturbative oscillations about them added) are playing dominant role in formation of the QCD vacuum. According to this scenario in the low energy region the coupling constant becomes frozen somewhere at the scale of average instanton size and the liquid (or gas) of (anti-)instantons randomly oriented in colour space and homogeneously distributed over 4-dimensional Euclidean space is an adequate ensemble to saturate the corresponding functional integral. The quarks are living in this background and its influence after all defines the observable quantities such as the gluon and chiral condensates, the dynamically generated quark mass the pion decay constant etc.

The pressing need in the profound study of quasi-classical approach at calculating the generating functional of QCD comes also from the recent developments of investigating a colour superconductivity which appears as a new phase of hadronic matter at very high density of quark/baryonic matter and low temperatures [2]. The analysis of quark condensate behaviour in the instanton liquid model (IL) [3] shows the corresponding phase transition takes place at a surprisingly small value of the critical density of quark/baryonic matter (order (or even less) of the quark density in normal nuclei). Applying the IL scheme devised in [4] which takes into account the medium collective excitations essentially allows to increase the critical density. Moreover, this density of phase transition may become so high that the Coulomb fields generated by quarks turn out comparable to the instanton ones due to very small interquark distances. Obviously, it puts in doubt the predictions of IL model which ignores the influence of external fields on the pseudo-particles and makes IL hardly applicable under such conditions.

Here we focus on studying the (anti-)instanton behaviour in the field of point-like Euclidean colour charge. Actually, this problem is one of the clue elements in constructing a more realistic version of IL, in particular, at high quark/baryonic densities. Besides, it can shed light on the challenging problem of colour field penetration into IL. We argue and deal with the field theory approach in quasi-classical approximation aiming to estimate the leading contributions. Apart from scrutinizing the terms of interaction between source and pseudo-particle which have been explored in the pioneering papers [1] and where its dipole nature has been argued, we are trying to clarify what happens to a pseudo-particle itself when an external field is available. Apparently, the problem of investigating the deformation fields is of great importance in this context. In spite of the rather impressive record of studying the pseudoparticle interactions (see, for example, [5], [6]) nowadays we are still far from declaring this problem resolved and transformed to the practical instrument. We formulate a further new approach to study the interactions of point-like solutions of field equations and suggest a new efficient way of approximate calculation of the functional integral for interacting pseudo-particles. In this way we construct the perturbative approach intrinsically related to the nontrivial topological solution itself analyzing the variations of solution parameters. In the case of pseudo-particles these configurations retain the bulk topological features and are sensitive to the presence of perturbative factors. At first sight, this rather routine excercise exhibits unexpectedly a wealth of possibilities and turns out very intersting even out of the IL context. We find out the pseudo-particle could be considered as a 'supplier' of various fields, in particular, a scalar field and colour vector and tensor fields. We expect the scalar field could be a good pretender to be an interaction carrier in the soft momentum region [7] whereas the vector field carrying the colour indecies could be responsible for the screening in the instanton medium. The paper is organized as follows. The motivation to consider the problem and supposed method of its treatment are discussed in Sec.1 The deliberation of deformation contributions to the total action of point-like source and instanton is given together with an approximate approach to calculate the pseudoparticle deformations in Sec.2 and the practical realization of this suggestion is demonstrated in Sec.3. In the fourth Section we present the analysis of numerical calculations and explore the generalized formulation of searching the deformations with imposed constraints. And eventually Section 5 is devoted to an analysis of the results obtained and their applications for calculating within the IL approach.

1 Optimizing the instanton configurations

Let us suppose the presence of immovable Euclidean colour point-like source of external field at the point $\boldsymbol{z}_e$. Its intensity is supposed to be $\boldsymbol{e}$ and an orientation along the third axis of colour (isotopic) space because for the sake of simplicity we limit ourselves here to dealing with $SU(2)$ group only. Then the current density takes the form $j^a_\mu = (\mathbf{0},\ e\ \delta^{a3}\delta(\boldsymbol{x}-\boldsymbol{z}_e))$ where $\boldsymbol{x}$, $\boldsymbol{z}_e$ are the 3-vectors $\mu = 1,2,3,4$ (as other Greek indecies) and $a = 1,2,3$ (as other Latin indecies). Fixing the location of point-like sources is gauge invariant unlike specifing their orientation in isotopic space. In order to

avoid such a stain [8] the static solutions of the Yang-Mills equations should be characterized by their energy and 'total' isospin because these quantities are gauge invariant in addition to being conserved. As known, the field originated by the particle source in 4d-space develops the cone-like shape with the edge being situated just at the particle creation. Then getting away from that point the field becomes fully developed in the area neighbouring it. In distant area where the field penetrates into the vacuum it should be described by the retarded solutions obeying, in particular, the Lorentz gauge. Here we are interested in studying the interaction in the neighbouring field area and the cylinder-symmetric Coulomb field might be its relevant image in 4d Euclidean space. Certainly, we skip any impact of the instanton field on the source of colour field although the self-consistent solution of the problem with non-abelian fields teaches about the possible change of the source orientation in colour space (see an example of two point-like colour charges [9]), see also [10]. However, bearing in mind that the instanton field is rather short range (the order of the PP size) we believe the source field is not reconstructed essentially on that scale (albeit at the end of this paper we will be well prepared to formulate the corresponding generalized equations for the field of point-like source).

The action for gauge field A^a_μ when an external source is available reads

$$S = \int dx \left(\frac{1}{4}\; G^a_{\mu\nu}\; G^a_{\mu\nu} + j^a_\mu \mathcal{A}^a_\mu\right) \;, \tag{1}$$

with the field strength defined in standard way

$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g\; \varepsilon^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \;, \tag{2}$$

where ε^{abc} is an entirely antisymmetric tensor and g a coupling constant for non-abelian field. Actually, the solution of the Yang-Mills equations corresponding to a single (anti-)instanton, for example, in the singular gauge is well defined and characterized by the coordinate of the PP centre z, the PP orientation in the colour space ω^{ab} and its size ρ, i.e.

$$A^a_\mu(x) = \frac{2}{g}\; \omega^{ab} \bar{\eta}_{b\mu\nu}\; \frac{\rho^2}{y^2+\rho^2}\; \frac{y_\nu}{y^2} \;, \tag{3}$$

where $y = x - z$ and $\bar{\eta}_{b\mu\nu}$ is the 't Hooft symbol (for anti-instanton $\bar{\eta} \to \eta$). When the distance between the point-like source and the instanton centre is large comparing to its size $\Delta \gg \rho$ ($\Delta = |\boldsymbol{\Delta}|$, $\boldsymbol{\Delta} = \boldsymbol{z} - \boldsymbol{z}_e$) the standard superposition

$$\mathcal{A}^a_\mu = B^a_\mu(x) + A^a_\mu(x) \;, \tag{4}$$

of point-like source field which for the sake of simplicity is considered in this paper in the simplest gauge only

$$B^a_\mu(x) = (\mathbf{0}, \delta^{a3}\; \varphi),\;\; \varphi = \frac{e}{4\pi}\; \frac{1}{|\boldsymbol{x} - \boldsymbol{z}_e|} \;, \tag{5}$$

and the (anti-)instanton field Eq.(3) is a pretty reasonable approximation of the true solution. The Lorentz gauge condition $\partial_\mu B^a_\mu = 0$ is valid for the retarded solution (as for the instanton field). In the region of a well developed field this condition is obeyed by the Coulomb solution Eq.(5) as well. We treat the potentials in their Euclidean forms but the following changes of the field and the charge of point-like source $B_0 \to iB_4$, $e \to -ie$ should be done at transition from the Minkowski space. Actually, last variable change is resulted from the corresponding transformations of spinor fields $\psi \to \hat{\psi}$, $\bar{\psi} \to -i\hat{\psi}^\dagger$, $\gamma_0 \to \gamma_4$ where the hatted spinors are Euclidean. Clearly, these Euclidean sources generate the fields of the same nature as ones we face taking into account gluon field quantum fluctuations a_{qu} around the classical instanton solutions $A = A_{inst} + a_{qu}$.

Searching the optimal PP configurations we fix its size and orientation in the isotopic space at a large distance from the source. But approaching the source (in the local vicinity of it) both are able

to vary as the functions of the PP centre coordinate, i.e. $\rho \to R(x,z)$, $\omega^{ab} \to \Omega^{ab}(x,z)$. Now with such substitutions in the superposition ansatz Eq.(4) we may formulate the variation problem of searching the deformation fields $R(x,z)$, $\Omega(x,z)$ which optimize the action Eq.(1) with the boundary conditions for these fields falling down at $\Delta \to \infty$. Finally, as an output we come to the sophisticated system of differential equations which can be analyzed numerically. However, such a scheme even treated perturbatively faces the principle difficulty because the coefficients of higher order derivatives of the deformation fields $R(x,z)$ and $\Omega(x,z)$ are strongly suppressed beyond PP owing to the almost singular behaviour of the (anti-)instanton solution. This fact essentially complicates the process of searching the appropriate solutions and, moreover, makes it practically impossible sometimes. At the same time this peculiarity of the PP solution behaviour prompts very natural approach to circumvent the difficulty mentioned. It implies applying the variational Ritz procedure (instead of the grid method) which in this particular case can be realized as the multipole expansion of the (anti-)instanton size, for example,

$$\begin{aligned} R_{in}(x,z) &= \rho + c_\mu\, y_\mu + c_{\mu\nu}\, y_\mu\, y_\nu + \dots\ , \qquad |y| \le L \\ R_{out}(x,z) &= \rho + d_\mu\, \frac{y_\mu}{y^2} + d_{\mu\nu}\, \frac{y_\mu}{y^2}\, \frac{y_\nu}{y^2} + \dots\ , \qquad |y| > L\ . \end{aligned} \tag{6}$$

Similar expressions could be written down for the (anti-)instanton orientation in the colour space $\Omega(x,z)$ where L is a certain parameter fixing the radius of sphere where the multipole expansion growing with the distance increasing should be changed for the decreasing one being a result of imposed deformation regularity constraint. Hoping to rely further on the perturbative analysis we have to calculate the coefficients $c_\mu, c_{\mu\nu}, \dots$ and $d_\mu, d_{\mu\nu}, \dots$ and the parameter L by minimizing the corresponding quadratic form resulting from Eq.(1), i.e. $\delta S = 0$, (being calculated up to the second order terms) together with the boundary conditions at infinity and the conditions of sewing $'in'$ and $'out'$ parameters on the sphere $|y| = L$. The final results allow us to conclude that the instanton field turns out to be so strong regularizator that we are allowed to deal with the fields R_{in} and Ω_{in} only at exploring the terms of the second order in small deviations from ρ and ω. One may neglect all the specifications coming from R_{out} and Ω_{out} which means formally to operate in the limit $L \gg \rho$. If one is interested in stationary cylindrically-symmetric picture in the 4-dimensional space the dependence on temporal coordinate x_4 should be excluded, i.e. the spatial indecies $1, 2, 3$ have to be retained in the coefficients of multipole expansion Eq.(6). Then it is clear the final result becomes dependent on the distance between three dimensinal components of the instanton centre and the source position which is denoted as Δ hoping it does not produce a mess. We would like to notice here that our condition to defined the deformations $\delta S = 0$ allows us to fix the major contributions to the exponent of the generating functional. In a sence we might consider this equation as the valley equation in the quasiclassical approximation. Then the higer order terms should reproduce loop corrections (would be quantum interactions).

Going to implement the program mentioned above let us consider the example of the dipole expansion terms (6) denoting them as

$$\delta\rho = \delta_\mu \rho\ y_\mu\ , \quad \delta\omega = \delta_\mu \omega\ y_\mu\ , \quad R = \rho + \delta\rho\ , \quad \Omega = \omega + \delta\omega\ . \tag{7}$$

Then for the crumpled instanton we have instead of (3) the following expansion in $\delta\rho$ up to the second order terms

$$A^a_\mu \simeq A^a_\mu(\Omega,\rho) + \frac{\partial A^a_\mu(\Omega,\rho)}{\partial R}\ \Delta R + \frac{\partial^2 A^a_\mu(\Omega,\rho)}{\partial R\ \partial R}\ \frac{(\Delta R)^2}{2}\ ,$$

here $\Delta R = R - \rho$. Using the superposition $B^a_\mu + A^a_\mu$ in Eq.(2) leads to the definition of $G^a_{\mu\nu}$ as

$$\begin{aligned} G^a_{\mu\nu} &= G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B)\ , \\ G^a_{\mu\nu}(A,B) &= g\ \varepsilon^{abc}(B^b_\mu A^c_\nu + A^b_\mu B^c_\nu)\ , \end{aligned} \tag{8}$$

with $G^a_{\mu\nu}(A)$ and $G^a_{\mu\nu}(B)$ in the standard form of Eq.(2) and the field strength squared looks then like the following sum

$$\begin{aligned} G^a_{\mu\nu}\, G^a_{\mu\nu} &= G^a_{\mu\nu}(B)\, G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A)\, G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B)\, G^a_{\mu\nu}(A,B) + \\ &+ 2\, G^a_{\mu\nu}(B)\, G^a_{\mu\nu}(A) + 2\, G^a_{\mu\nu}(B)\, G^a_{\mu\nu}(A,B) + 2\, G^a_{\mu\nu}(A)\, G^a_{\mu\nu}(A,B)\ . \end{aligned} \tag{9}$$

One should not envisage this complicated expression tractable in the approximation of weak external field because of the strongly singular nature of instanton solution and point-like source field. Besides, the large number of interacting terms essentially contributing does not allow one to be based on their physical meaning only (as in [11]) in further analysis. Now let us note the non-zero strength component of point-like source field reads

$$G^a_{4i}(B) = \frac{e}{4\pi}\, \delta^{a3}\, \frac{y_i + \Delta_i}{|\boldsymbol{y} + \boldsymbol{\Delta}|^3}\ .$$

whereas the field strength generated by the instanton is presented as

$$G^a_{\mu\nu}(A) = \hat{G}^a_{\mu\nu}(A) + \Delta G^a_{\mu\nu}(A)\ , \tag{10}$$

and the first term here looks like instanton field strength but with the varying parameters

$$\hat{G}^a_{\mu\nu}(A) = -\frac{4}{g}\, \Omega^{ak} M_{\mu\alpha} M_{\nu\beta}\, \bar{\eta}_{k\alpha\beta}\, \frac{R^2}{(y^2+R^2)^2}\ ,$$

where $M_{\mu\alpha} = \delta_{\mu\alpha} - 2\,\frac{y_\mu y_\alpha}{y^2}$, and the correction term is given by

$$\begin{aligned} \Delta G^a_{\mu\nu}(A) &= \frac{\partial A^a_\nu(\Omega,R)}{\partial R}\, \frac{\partial R}{\partial y_\mu} - \frac{\partial A^a_\mu(\Omega,R)}{\partial R}\, \frac{\partial R}{\partial y_\nu} + \frac{\partial A^a_\nu(\Omega,R)}{\partial \Omega^{bc}}\, \frac{\partial \Omega^{bc}}{\partial y_\mu} - \frac{\partial A^a_\mu(\Omega,R)}{\partial \Omega^{bc}}\, \frac{\partial \Omega^{bc}}{\partial y_\nu} + \\ &+ \frac{4}{g} [\varepsilon^{abc}(\omega^{bm}\delta\omega^{cn} - \omega^{bn}\delta\omega^{cm} + \delta\omega^{bm}\omega^{cn}) - \delta\omega^{ak}\varepsilon^{kmn}]\, \bar{\eta}_{m\mu\alpha}\bar{\eta}_{n\nu\beta}\, \frac{R^4}{(y^2+R^2)^2}\, \frac{y_\alpha y_\beta}{y^4}\ . \end{aligned}$$

Finally, the interfering term takes the following form

$$G^a_{4i}(A,B) = 2\, \frac{e}{4\pi}\, \varepsilon^{a3c}\, \Omega^{ck}\, \bar{\eta}_{ki\alpha}\, \frac{y_\alpha}{y^2}\, \frac{R^2}{(y^2+R^2)}\, \frac{1}{|\boldsymbol{y} + \boldsymbol{\Delta}|}\ .$$

The other terms do not contribute to $G^a_{\mu\nu}(A,B)$ because of the particular choice of gauge. Later we will develop the perturbative approach for this part of strength expanding it in $\delta\rho\ \ \delta\omega$ up to the second order terms.

2 Classifying the deformation contributions

The first term of Eq.(9) is certainly out of our interest here and calculating the action with the second term included gives the correction to single instanton action. The quadratic form of kinetic energy type hinges upon the deformation fields

$$\begin{aligned} S_{kin} &= \int dx\, \frac{1}{4}\, G^a_{\mu\nu}(A) G^a_{\mu\nu}(A) - \beta = \frac{\kappa}{2}\, (\delta_\mu\rho)^2 + \frac{\sigma^{kl}_{\mu\nu}}{2}\, \delta_\mu\omega^{ak}\delta_\nu\omega^{al} + \frac{\sigma_3}{2}\, \varepsilon^{kmn}\varepsilon^{abc}\omega^{bm}\delta_\mu\omega^{ak}\delta_\mu\omega^{cn} + \\ &+ \frac{\sigma_4}{2}\, \bar{\eta}_{m\mu\nu}\varepsilon^{abc}(\omega^{bm}\delta_\nu\omega^{cn} - \omega^{bn}\delta_\nu\omega^{cm})\delta_\mu\omega^{an} + \frac{\sigma_5}{2}\, \omega^{am}\omega^{cn}\delta_\mu\omega^{am}\delta_\mu\omega^{cn} + \tilde{v}^{al}_{\mu\nu}\, \delta_\mu\omega^{al}\delta_\nu\rho, \end{aligned} \tag{11}$$

with the tensor coefficients

$$
\begin{aligned}
&\kappa = \frac{9}{10}\,\beta\ ,\quad \sigma^{kl}_{\mu\nu} = \sigma_1\ \delta_{\mu\nu}\delta_{kl} + \sigma_2\ \varepsilon_{kln}\ \bar{\eta}_{n\mu\nu}\ ,\quad \widetilde{v}^{al}_{\mu\nu} = \omega^{ak} v^{kl}_{\mu\nu}\ ,\quad v^{kl}_{\mu\nu} = v_1\ \delta_{\mu\nu}\delta_{kl} + v_2\ \varepsilon_{kln}\ \bar{\eta}_{n\mu\nu}\ ,\\
&\sigma_1 = \frac{23}{24}\,\beta\,\rho^2\ ,\quad \sigma_2 = -\frac{3}{8}\,\beta\,\rho^2\ ,\quad \sigma_3 = -2v_1\ \rho\ ,\\
&\sigma_4 = 2v_2\ \rho\ ,\quad \sigma_5 = \frac{4}{3}\,v_2\ \rho\ ,\quad v_1 = \frac{7}{24}\,\beta\,\rho\ ,\quad v_2 = \frac{1}{8}\,\beta\,\rho\ ,
\end{aligned}
\tag{12}
$$

where $\beta = \frac{8\pi^2}{g^2}$ is the single (anti-)instanton action. These contributions are regulated by instanton itself and independent of the field B as well as the distance between the pseudo-particle center and the source. In a sense they characterize the PP compliantness as for its shape changes and an orientation in colour space. Here is an appropriate place to notice that dealing with the crumpled configurations characterized by the parameters (7) within the perturbative approximation and in the limit $L \to \infty$ (see (6)) presupposes a lot of singular terms available. Fortunately, the natural regularization pops in when those bad terms are multiplied by the instanton tensor $G^a_{\mu\nu}(\rho,\omega)$ (see below (22)). It is interesting to notice the operator similar to Eq.(11) emerges when one explores the fluctuation fields in the proximity of the particular classical solution $A = A_{cl} + a_{qu}$ and the action acquires an additional quadratic term

$$
S = S(A_{cl}) + \int dx\ a_{qu}\ \mathcal{L}(A_{cl})\ a_{qu} = S(A_{cl}) + \sum_k \lambda_k\ \xi_k^2\ ,
\tag{13}
$$

where λ_k are the eigenvalues of the operator $\mathcal{L}$ and ξ_k presents the coefficients of the expansion of field $a_{qu} = \sum_k \xi_k \psi_k$ into the entire set of orthogonal eigenfunctions ψ_k of this operator. In view of that generally Eq.(11) delineates the parametric dependence of eigenvalues and eigenfunctions of the operator $\mathcal{L}$ in the accepted approximation. In our particular case it means dependence on the expansion coefficients in Eq.(6) $\left(\frac{\partial\lambda_k}{\partial c_\mu}, \frac{\partial\psi_k}{\partial c_\mu}, \dots\right)$. Apparently, we gain the possibility to take into account the contribution of regular component originated by external field in the functional integral instead of dealing with poorly defined Green function of instanton.

Calculating Eq.(12) and some subsequent ones we used the following identities valid for $SU(2)$-group

$$
\omega^{ab}\omega^{ac} = \delta^{bc}\ ,\quad \varepsilon^{abc}\omega^{a\alpha}\omega^{b\beta}\omega^{c\gamma} = \varepsilon^{\alpha\beta\gamma}\ .
\tag{14}
$$

It is practical for analysing the additional term to the single instanton action to present it as a sum of two components

$$
S - \beta = S_{kin} + S_{int}\ .
\tag{15}
$$

where S_{kin} is the term of kinetic energy type and S_{int} is the interacting term (of course, the singular contribution of the point-like source to the action is ignored). The interacting term S_{int} may be decomposed into the following sum

$$
S_{int} = S^{\kappa}_{int} + S^{d}_{int} + S^{\delta}_{int}\ .
\tag{16}
$$

and the physical meaning of each term here becomes clear from further calculations. We quote here the final result only ommitting the routine calculations which are not very inventive and require the good patience mainly.

2.1 Quadratic terms depending on the distance from source

As the first term of Eq.(16) we mean the quadratic terms in $\delta\rho$, $\delta\omega$ wholesale which describe the dependence of kinetic coefficients on Δ:

$$
S^{\kappa}_{int} = \frac{1}{g}\frac{e}{4\pi}\ \omega^{3k}\bar{\eta}_{k4l}\ \Sigma_{lij}\ \delta_i\rho\ \delta_j\rho + \left(\frac{e}{4\pi}\right)^2 (\delta_{kl} - \omega^{3k}\omega^{3l})(\delta_{kl}\ Q_{ij} - \bar{\eta}_{k4m}\bar{\eta}_{l4n}\ R_{mnij})\ \delta_i\rho\ \delta_j\rho\ -
$$

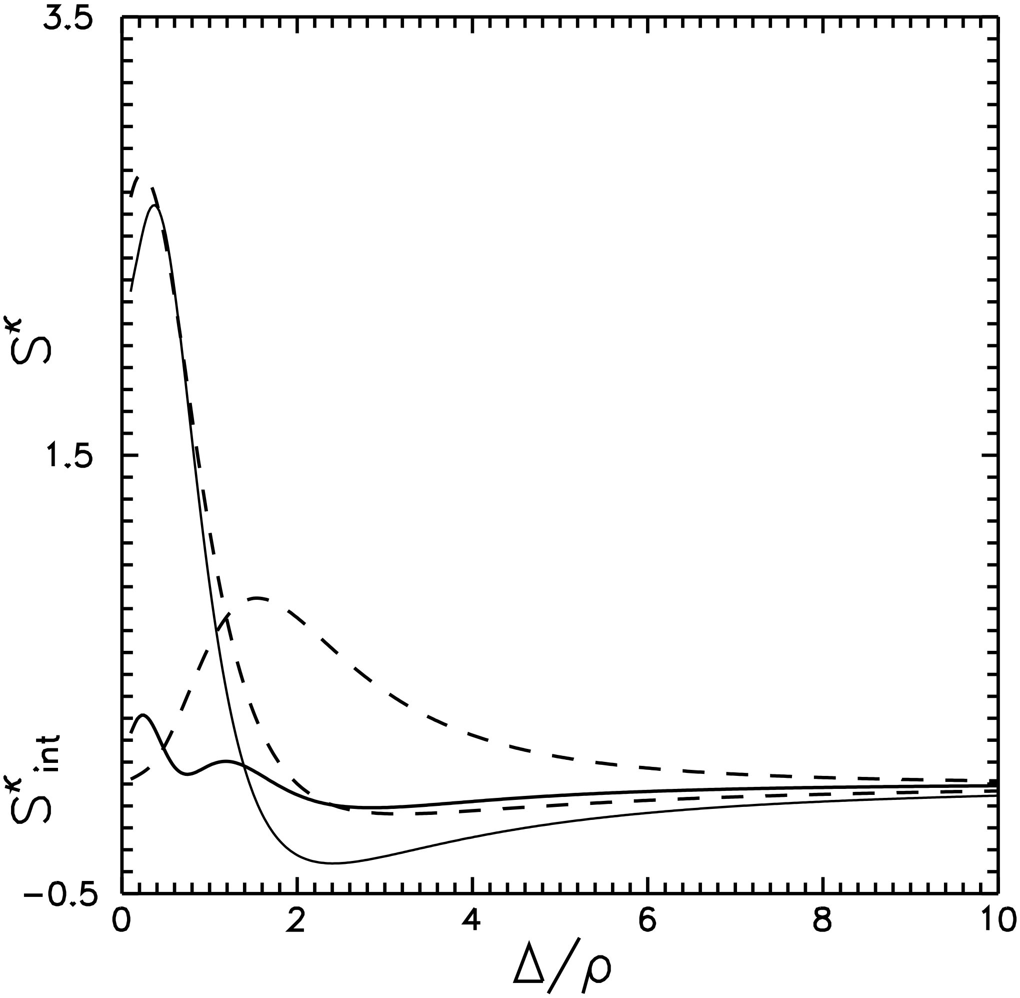

Figure 1: The comparison of contributions S_{kin} and S^{κ}_{int} as the functions of the path along x axis. The instanton action is $\beta = 18$ and the source intensity is taken as $e = g$. The rotation matrix ω^{ak} is defined by the parameters $\boldsymbol{n} = (0, 1, 0)$, $\phi = \pi/2$ (see Eq.(23)). Two upper curves (in the vicinity of coordinate origin) demonstrate the S_{kin} behaviour whereas two lower ones demonstrate the S^{κ}_{int} behaviour. The solid curves correspond to the instanton and the dashed ones correspond to the anti-instanton. The source e is oriented along the third axis in the isotopic space.

$$
\begin{aligned}
&-\frac{1}{g}\frac{e}{4\pi}\ \bar\eta_{k4m}(\bar\eta_{lin}\ S_{jmn} + \bar\eta_{lmn}\ T_{nij})\varepsilon^{a3b}\ \delta_i\omega^{ak}\ \delta_j\omega^{bl} - \frac{1}{g}\frac{e}{4\pi}\ \varepsilon^{a3b}\bar\eta_{k4m}\bar\eta_{lnc}\ Z_{mcijn}\ \delta_i\omega^{ak}\delta_j\omega^{bl} + \\
&+\frac{1}{g}\frac{e}{4\pi}\ \varepsilon^{a3c}\bar\eta_{kl\alpha}\bar\eta_{m4\beta}\bar\eta_{nl\gamma}\ \Phi_{\alpha\beta\gamma ji}\ [\varepsilon^{abd}(\omega^{bm}\delta_i\omega^{dn} - \omega^{bn}\delta_i\omega^{dm}) - \varepsilon^{dmn}\delta_i\omega^{ad}]\delta_j\omega^{ck} + \\
&+\left(\frac{e}{4\pi}\right)^2(\delta_{kl}\ U_{ij} - \bar\eta_{k4m}\bar\eta_{l4n}\ V_{ijmn})(\delta_i\omega^{ak}\ \delta_j\omega^{al} - \delta_i\omega^{3k}\ \delta_j\omega^{3l}) + \frac{1}{g}\frac{e}{4\pi}\ \bar\eta_{k4l}\ \Psi_{ilj}\ \delta_i\omega^{3k}\ \delta_j\rho + \\
&+\frac{1}{g}\frac{e}{4\pi}[(\bar\eta_{l4m}\bar\eta_{kmn} - \bar\eta_{k4m}\bar\eta_{lmn})W_{ijn} + (\bar\eta_{l4m}\bar\eta_{kjn} - \bar\eta_{k4m}\bar\eta_{ljn})X_{mni} + 2\bar\eta_{l4m}\bar\eta_{kin}X_{mnj}]\omega^{ak}\varepsilon^{a3b}\delta_i\omega^{bl}\delta_j\rho + \\
&+\frac{1}{g}\frac{e}{4\pi}\ \varepsilon^{a3c}\omega^{ck}\bar\eta_{kl\alpha}\bar\eta_{m4\beta}\bar\eta_{nl\gamma}\ \Theta_{\alpha i\beta\gamma j}\ [\varepsilon^{abd}(\omega^{bm}\delta_i\omega^{dn} - \omega^{bn}\delta_i\omega^{dm}) - \varepsilon^{dmn}\delta_i\omega^{ad}]\ \delta_j\rho + \\
&+\frac{1}{g}\frac{e}{4\pi}\ \bar\eta_{c4k}\bar\eta_{dln}\ \Xi_{knijl}\ [\varepsilon^{3ab}(\omega^{ac}\delta_i\omega^{bd} - \omega^{ad}\delta_i\omega^{bc}) - \varepsilon^{acd}\delta_i\omega^{3a}]\ \delta_j\rho\ .
\end{aligned}
\tag{17}
$$

Here we are studying an impact of fully developed Coulomb field on an instanton and that is why the above formula includes the spatial indecies only of the deformation fields $\delta_\mu\rho$, $\delta_\mu\omega$. The overt but cumbersome representations of the tensor components are given in Appendix. In this paragraph we are interested in their asymptotic values at $\Delta \to \infty$ only and it is convenient for further analysis

to use the dimensionless coordinates according to the following change $\Delta_i/\rho \to \Delta_i$. The terms of $O((\delta\rho)^2)$ are presented by three tensors. One of those is the third rank tensor

$$\Sigma_{ijk} = \delta_{ij}\hat{\Delta}_k\ \Sigma_1 + \delta_{ik}\hat{\Delta}_j\ \Sigma_2 + \delta_{jk}\hat{\Delta}_i\ \Sigma_3 + \hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k\ \Sigma_4\ ,$$
$$\Sigma_1 \simeq -2\pi^2\ ,\quad \Sigma_2 \simeq 2\pi^2\ ,\quad \Sigma_3 \simeq \frac{49\pi^2}{3}\frac{\ln\Delta}{\Delta^2}\ ,\quad \Sigma_4 \simeq 0\ ,$$

where $\hat{\Delta}_i = \frac{\Delta_i}{\Delta}$ is the unit vector; another one is the entirely symmetric tensor of the second rank

$$Q_{ij} = \delta_{ij}\ Q_1 + \hat{\Delta}_i\hat{\Delta}_j\ Q_2\ , \tag{18}$$

with the components

$$Q_1 \simeq 6\pi^2\frac{\ln\Delta}{\Delta^2}\ ,\quad Q_2 \simeq 0\ ;$$

and eventually the entirely symmetric tensor of the fourth rank

$$\begin{aligned} R_{ijkl} &= (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\ R_1 + (\delta_{ij}\hat{\Delta}_k\hat{\Delta}_l + \delta_{ik}\hat{\Delta}_j\hat{\Delta}_l + \delta_{il}\hat{\Delta}_j\hat{\Delta}_k + \\ &+ \delta_{kl}\hat{\Delta}_i\hat{\Delta}_j + \delta_{jl}\hat{\Delta}_i\hat{\Delta}_k + \delta_{jk}\hat{\Delta}_i\hat{\Delta}_l)\ R_2 + \hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k\hat{\Delta}_l\ R_3, \end{aligned} \tag{19}$$

where

$$R_1 \simeq \pi^2\frac{\ln\Delta}{\Delta^2}\ ,\quad R_2 \simeq 0\ ,\quad R_3 \simeq 0\ .$$

If the particular component of any tensor is small comparing to the leading asymptotic term and is out of discussion we imply zero value for it and don't show its asymptotic form. All the tensor-functions, excluding the components Σ_1, Σ_2 of Σ_{ijk}, are decreasing with the distance increasing. An appearence of these nonzero asymptotic values at $\Delta \to \infty$ results from the contribution of interference term originated by the product of source field strength $G^a_{4i}(B)$ and the $G^a_{4i}(A)$ component of second order in $(\delta\rho)^2$. Such behaviour is generated by the substitution where the parameter fixing above mentioned radius of multipole expansion $L \to \infty$.

The coefficients of the $O((\delta\omega)^2)$ terms are constructed by two entirely symmetric tensors S and T of the third order. They are

$$S_{ijk} = (\delta_{ij}\hat{\Delta}_k + \delta_{ik}\hat{\Delta}_j + \delta_{jk}\ \hat{\Delta}_i)\ S_1 + \hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k\ S_2\ , \tag{20}$$

with the components

$$S_1 \simeq -\frac{\pi^2}{3}\frac{\ln\Delta}{\Delta^2}\ ,\quad S_2 \simeq 0\ ,$$

and the tensor T is analogous to Eq.(20) with similarly looking components

$$T_1 \simeq -\frac{\pi^2}{2}\frac{\ln\Delta}{\Delta^2}\ ,\quad T_2 \simeq 0\ .$$

Besides, Eq.(17)contains the fifth rank tensor Z in the following form

$$\begin{aligned} Z_{ijklm} &= [\hat{\Delta}_i(\delta_{jk}\delta_{lm} + \delta_{jl}\delta_{km} + \delta_{jm}\delta_{kl}) + \hat{\Delta}_j(\delta_{ik}\delta_{lm} + \delta_{il}\delta_{jm} + \delta_{im}\delta_{kl}) + \\ &+ \hat{\Delta}_k(\delta_{ij}\delta_{lm} + \delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) + \hat{\Delta}_l(\delta_{ij}\delta_{km} + \delta_{ik}\delta_{jm} + \delta_{im}\delta_{jk})]\ Z_1 + \\ &+ \hat{\Delta}_m(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\ Z_2 + \hat{\Delta}_m[\delta_{ij}\hat{\Delta}_k\hat{\Delta}_l + \delta_{ik}\hat{\Delta}_j\hat{\Delta}_l + \delta_{il}\hat{\Delta}_j\hat{\Delta}_k + \\ &+ \delta_{jk}\hat{\Delta}_i\hat{\Delta}_l + \delta_{jl}\hat{\Delta}_i\hat{\Delta}_k + \delta_{kl}\hat{\Delta}_i\hat{\Delta}_j]\ Z_3 + [\delta_{im}\hat{\Delta}_j\hat{\Delta}_k\hat{\Delta}_l + \\ &+ \delta_{jm}\hat{\Delta}_i\hat{\Delta}_k\hat{\Delta}_l + \delta_{km}\hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_l + \delta_{lm}\hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k]\ Z_4 + \hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k\hat{\Delta}_l\hat{\Delta}_m\ Z_5\ , \end{aligned} \tag{21}$$

their components have no singularities and are going to zero in the limit $\Delta \to \infty$. There exists the contribution of entirely symmetric tensor of the fifth rank Φ in Eq.(17) which has the form analogous

to Z with the components $\Phi_1 = \Phi_2$, $\Phi_3 = \Phi_4$. In the Appendix below two first components are denoted by Φ_1, another two as Φ_2 and, finally, the component Φ_5 is denoted by Φ_3. Unlikely the tensor Z the component of tensor Φ having two time indecies ($\Phi_{4\beta 4ji}$) provides the terms of kinetic energy type with non-zero contribution. We demonstrate two nontrivial components of construction of the S_{ijk}-type (Eq.(20)) which are denoted as $\Phi^{44}_{1,2}$ in Appendix. As tensor Z the tensor Φ does not develop the singularities in the limit $\Delta \to \infty$. The other terms of Eq.(17) are given by the entirely symmetric tensor of the second rank U similar to Eq.(18) with the components

$$U_1 \simeq \pi^2 \, \frac{\ln \Delta}{\Delta^2} \ , \quad U_2 \simeq 0 \ ;$$

and the entirely symmetric tensor of the fourth rank V of the Eq.(19) type with the components

$$V_1 \simeq -\frac{\pi^2}{4} \frac{\ln \Delta}{\Delta^2} \ , \quad V_2 \simeq 0 \ , \quad V_3 \simeq 0 \ .$$

In distinction with the dilatation term Σ in the contribution of kinetic energy type Eq.(17) the term generated by the rotations in the isotopic space has no singular contributions (they compensate each other at the stage of intermediate calculations) and in output the tensor components are decreasing with the distance increasing.

For the mixed contributions of $O(\delta\rho \ \delta\omega)$ we have the tensor with the component approaching a constant value again

$$\Psi_{ijk} = (\delta_{ij}\hat{\Delta}_k + \delta_{jk}\hat{\Delta}_i) \ \Psi_1 + \delta_{ik}\hat{\Delta}_j \ \Psi_2 + \hat{\Delta}_i\hat{\Delta}_j\hat{\Delta}_k \ \Psi_3 \ ,$$
$$\Psi_1 \simeq 0 \ , \quad \Psi_2 \simeq 0 \ , \quad \Psi_3 \simeq 4\pi^2 \ ;$$

(the singularity in the third component is related to the term jA of the action Eq.(1)), and two other entirely symmetric tensors W and X of the third rank (of the Eq.(20) type again) with the following components

$$W_1 \simeq -2\pi^2 \frac{\ln \Delta}{\Delta^2} \ , \quad W_2 \simeq 0 \ , \quad X_1 \simeq -\frac{4\pi^2}{3} \frac{\ln \Delta}{\Delta^2} \ , \quad X_2 \simeq 0 \ .$$

The entirely symmetric tensor of the fifth rank Θ available in Eq.(17) is constructed to be similar to Φ with the nontrivial components $\Theta_{1,2,3}$, $\Theta^{44}_{1,2} = \Theta_{4i\beta 4j}$. And, finally, the fifth rank tensor Ξ takes the form of Eq.(21). Both tensors Θ and Ξ have no the singularities in the limit $\Delta \to \infty$.

Now collecting the asymptotic values obtained all together we have for the additional contribution to the kinetic energy at $\Delta \to \infty$ (within the logarithmic precision) the following equation

$$S^{\kappa}_{int} \to \frac{1}{g} \frac{e}{4\pi} \ 2\pi^2 \omega^{3k} \bar{\eta}_{k4l} \ (-\hat{\Delta}_j \delta_{li} + \hat{\Delta}_i \delta_{lj}) \ \delta_i \rho \ \delta_j \rho = 0 \ , \tag{22}$$

which implies that any singularities available in the components of tensors do not produce any impact on the observables and, hence, the simplified procedure of the Ritz method with the parameter $L \to \infty$ is quite applicable.

The complete analysis of the PP deformations including the contribution of the term S^{κ}_{int} Eq.(17) may be cause for concern. However, the approximate procedure of searching the optimal deformations could be suggested just for the parameters interesting for applications in the IL model ($\beta = \frac{8\pi^2}{g^2} \simeq 12 \div 18$) [12]. Let us address Fig.1 where the contributions S_{kin} and S^{κ}_{int} for the single instanton are compared as functions of the path along the x-axis. We suggest for the instanton action $\beta = 18$ and the point-like souce intensity is taken as $e = g$. As to the rotation matrix it is defined by the vector $\mathbf{n} = (0, 1, 0)$ which fixes the PP rotation in the isotpic space to the angle φ in the following form

$$\omega^{ab} = n_a n_b + P_{ab} \cos\varphi - \varepsilon_{abc} \ n_c \sin\varphi \ , \tag{23}$$

where $P_{ab} = \delta_{ab} - n_a n_b$ is the appropriate projection operator. In the example under discussion it is taken $\varphi = \pi/2$. If one neglects the contribution of S^{κ}_{int} (comparing it to the contribution of S_{kin}) the deformations $\delta\rho$, $\delta\omega$ could be calculated in the true form (see Eq.(32) below). It is clear from Fig.1 the application of this approximation is well justified at small distances from the point-like source for both the instanton and the anti-instanton. The contribution S^{κ}_{int} for the anti-instanton in the region of instanton size order is rather large formally but in this case the contribution of the term of kinetic energy type is small compared to S^{δ}_{int} and the solution may be also used for the rough quantatative estimate. In what follows all the figures are obtained for the same parameters of rotation matrix and the same path as in Fig.1.

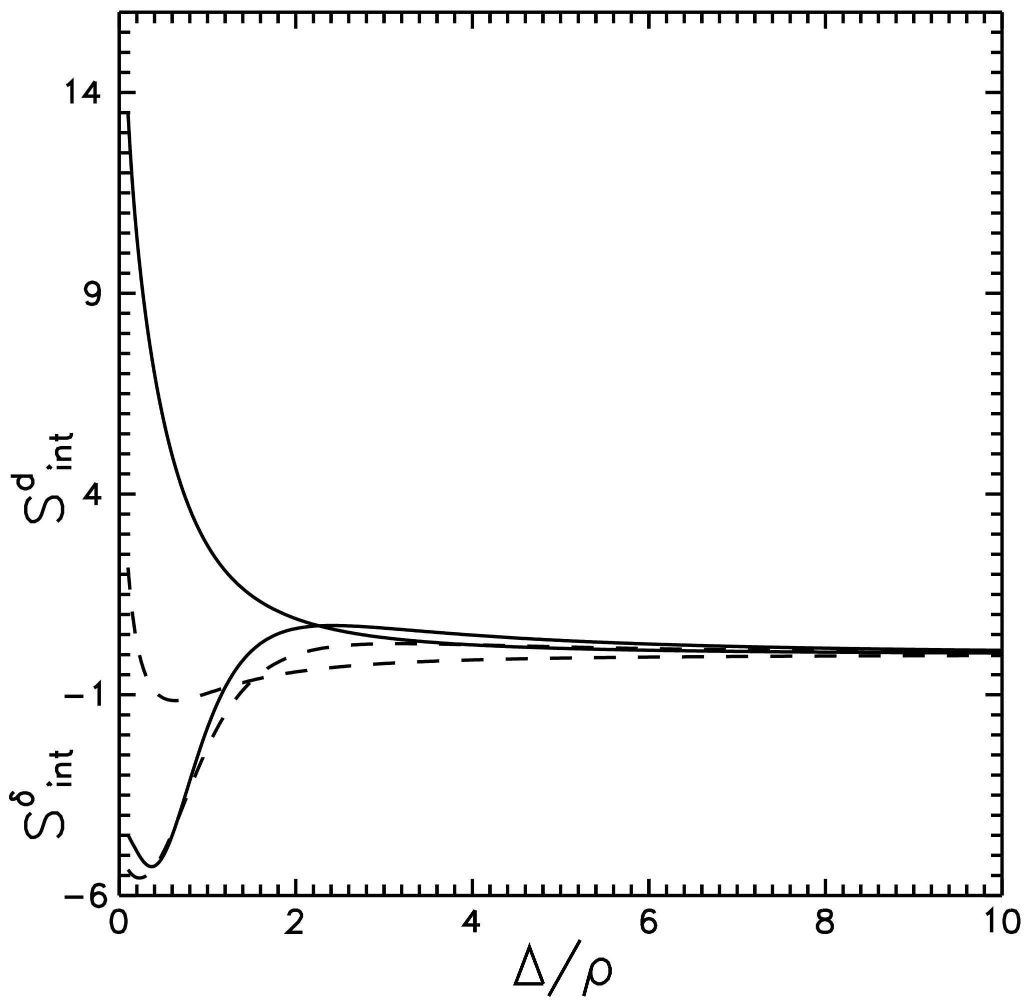

Figure 2: Simile of the contributions to S_{int}. The path and parameters which define the rotation matrix are the same as in Fig. 1. Two upper curves show S^{d}_{int} and two lower ones demonstrate S^{δ}_{int}. The solid curves correspond to an instanton whereas both dashed curves correspond to an anti-instanton.

2.2 Analysing interaction of the undeformed instanton

Here we are going to analyze the other interaction terms in Eq.(16). The second term there hinges directly on the asymptotic value of instanton size ρ and orientation ω^{ak} and describes the interaction of undeformed instanton with the point-like source

$$S^{d}_{int} = \frac{1}{g}\,\frac{e}{4\pi}\;\bar{\eta}_{k4i}\;\omega^{3k} I_i + \left(\frac{e}{4\pi}\right)^2 J + \left(\frac{e}{4\pi}\right)^2 K_{kl}\;\omega^{3k}\omega^{3l}\;, \tag{24}$$

It is interesting to notice the first term with the coefficient

$$I_i = 8\ \pi^2\ \Delta_i\ (D^{-1} - \Delta^{-1})$$

where $D^2 = \Delta^2 + 1$ is entirely generated by the term jA of initial action Eq.(1). And the contributions of chromoelectric and chromomagnetic fields to S^d_{int} in two terms

$$2\ G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(A) + 2\ G^a_{\mu\nu}(A)\ G^a_{\mu\nu}(A,B)\ , \tag{25}$$

after integrating are cancelled. If it occurs the instanton field is a weak perturbation of the Coulomb background (just the CDG approximation) then the first term becomes dominant and one can obtain

$$\frac{1}{4}\int dx\ 2\ G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(A) = \frac{e}{4\pi}\ \frac{8\pi^2}{g}\ \bar\eta_{k4i}\ \omega^{3k}\ \frac{\Delta_i}{\Delta^2}\ (2\ D^3 - 2\ \Delta^3 - 3D + D^{-1})\ .$$

Turning to the limit $\Delta \to \infty$ we get the well known result for the dipole interaction of an instanton with an external field

$$\frac{1}{4}\int dx\ 2\ G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(A) \to \frac{2\pi^2}{g}\ \bar\eta_{k4i}\ \omega^{3k}\ E_i\ ,$$

if the transition to the Minkowski space is performed and the dimensional variables are restored (see remarks after Eq.(5)). The vector E_i above presents the source field in the following form

$$E_i = \frac{e}{4\pi}\ \frac{\Delta_i}{\Delta^3}\ .$$

The function

$$J = 2\ \int dy\ \frac{2\ y^2 - \boldsymbol{y}^2}{y^4\ (y^2+1)^2\ |\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ ,$$

and tensor

$$K_{kl} = 2\ \int dy\ \frac{y_k y_l}{y^4}\ \frac{1}{(y^2+1)^2\ |\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ ,$$

cannot be integrated in the elementary functions although numerically can be analysed comprehensively. Their asymptotic values at $\Delta \to \infty$ look simply

$$J \simeq \frac{5\pi^2}{2}\frac{1}{\Delta^2}\ ,$$

and for the components of the second rank tensor Eq.(18) as

$$K_1 \simeq \frac{\pi^2}{2}\frac{1}{\Delta^2}\ ,\quad K_2 \simeq 0\ .$$

Now taking for the action of single instanton, for example, $\beta = 18$ and for the source intensity $e = g$ we find out the contribution of the terms proportional e^2 in Eq.(24) at small distances ($\Delta \leq 1$) is noticeably larger than the first term contribution. The behaviour of the curve corresponding to S^d_{int} in Fig.2 allows us to make such a conclusion. If the contribution of the first dipole-like term in Eq.(24) is the leading one the dashed curve corresponding to the anti-instanton should run completely in the region of negative values. Further on in this paper we face another interesting result related to the slow decreasing of these functions while applied to IL.

2.3 Linear deformations

Apparently, the interaction terms which allow us to detach the optimal deformations of PP are of major interest in this paper. Those terms are just linear in $\delta\rho$ and $\delta\omega$ and could be given in the following form

$$S^{\delta}_{int} = \Lambda_n\ \delta_n\rho + M^{ak}_n\ \delta_n\omega^{ak}\ , \tag{26}$$

where

$$\Lambda_n = \frac{1}{g}\ \frac{e}{4\pi}\ \bar\eta_{k4i}\ \omega^{3k}\ A_{in} + \left(\frac{e}{4\pi}\right)^2\ B_n + \left(\frac{e}{4\pi}\right)^2\ \omega^{3k}\omega^{3l}\ C_{knl}\ . \tag{27}$$

The tensor components are cited in Appendix and here we give their asymptotic values at large distances. For example, for the second rank tensor A which is similar Eq.(18) these values appear as

$$A_1 \simeq -2\pi^2\frac{1}{\Delta}\ ,\quad A_2 \simeq 16\pi^2\frac{1}{\Delta}\ .$$

The vector-function B_i spanned on the unit vector $\hat\Delta$ has the form

$$B_i = \hat\Delta_i\ B\ ,\quad B \simeq -\frac{28\pi^2}{3}\frac{\ln\Delta}{\Delta^3}$$

and two components of the third rank tensor C similar Eq.(20) are

$$C_1 \simeq -\frac{4\pi^2}{3}\frac{\ln\Delta}{\Delta^3}\ ,\quad C_2 \simeq 0\ .$$

Finally, the term in Eq.(26) related to the rotations in isotopic space is defined by the following tensor-functions

$$\begin{aligned} M^{ak}_n &= \frac{1}{g}\ \frac{e}{4\pi}\bar\eta_{k4i}\ D_{in}\ \delta^{a3} - \frac{1}{g}\ \frac{e}{4\pi}\ \varepsilon^{a3c}\ \omega^{cl}(\bar\eta_{k4m}\bar\eta_{lni}\ E_{mi} + \bar\eta_{k4j}\bar\eta_{ljm}\ F_{mn}) + \\ &+ \frac{1}{g}\ \frac{e}{4\pi}\ [\varepsilon^{a3b}\ \omega^{bc}(\bar\eta_{c4j}\bar\eta_{kil} - \bar\eta_{k4j}\bar\eta_{cil}) - \delta^{a3}\varepsilon^{kbc}\bar\eta_{b4j}\bar\eta_{cil}]\ H_{jlni} + \\ &+ \frac{1}{g}\ \frac{e}{4\pi}\ [(\omega^{ab}\omega^{3c} - \delta^{3a}\delta^{bc})\delta^{kl} - (\omega^{ab}\omega^{3l} - \delta^{3a}\delta^{lb})\delta^{kc} - \varepsilon^{a3d}\omega^{db}\varepsilon^{kcl}]\ \bar\eta_{bi\alpha}\bar\eta_{c4\gamma}\bar\eta_{li\delta}\ Y_{\alpha n\gamma\delta} + \\ &+ \left(\frac{e}{4\pi}\right)^2 (\omega^{al} - \omega^{3l}\delta^{a3})(O_n\delta_{kl} - P_{lkn})\ . \end{aligned} \tag{28}$$

Then the corresponding asymptotic values when $\Delta \to \infty$ for two components of the D tensor similar to Eq.(20) are

$$D_1 \simeq 0\ ,\quad D_2 \simeq 8\pi^2\frac{1}{\Delta}\ .$$

For the components of tensors E and F similar to Eq.(20) we have

$$E_1 \simeq \pi^2\frac{1}{\Delta}\ ,\quad E_2 \simeq 0\ ,\quad F_1 \simeq 2\pi^2\frac{1}{\Delta}\ ,\quad F_2 \simeq 0\ .$$

There is nothing special to get for the vector function

$$O_i = \hat\Delta_i\ O\ ,\quad O \simeq -4\pi^2\frac{\ln\Delta}{\Delta^3}$$

and for the components of the third rank tensor P similar to Eq.(20)

$$P_1 \simeq -\frac{2\pi^2}{3}\frac{1}{\Delta}\ ,\quad P_2 \simeq -\frac{8\pi^2}{3}\frac{1}{\Delta}\ .$$

The fourth rank tensor H reads

$$\begin{aligned} H_{ijkl} &= (\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\ H_1+(\delta_{ij}\hat\Delta_k\hat\Delta_l+\delta_{ik}\hat\Delta_j\hat\Delta_l+\delta_{jk}\hat\Delta_i\hat\Delta_l)\ H_2+ \\ &+ (\delta_{il}\hat\Delta_j\hat\Delta_k+\delta_{kl}\hat\Delta_i\hat\Delta_j+\delta_{jl}\hat\Delta_i\hat\Delta_k)\ H_3+\hat\Delta_i\hat\Delta_j\hat\Delta_k\hat\Delta_l\ H_4\ , \end{aligned}$$

with the following asymptotics of its components

$$H_1 \to -\frac{13}{36}\frac{\pi^2}{\Delta^3}\ ,\quad H_2 \to -\frac{53}{12}\frac{\pi^2}{\Delta^3}\ ,\quad H_3 \to \frac{1}{2}\frac{\pi^2}{\Delta^3}\ ,\quad H_4 \to -\frac{5}{2}\frac{\pi^2}{\Delta^3}\ .$$

For the space components of entirely symmetric tensor of the fourth rank Y similar to Eq.(19) we receive

$$Y_1 \to \frac{1}{6}\frac{\pi^2}{\Delta}\ ,\quad Y_2 \to 0\ ,\quad Y_3 \to 0\ .$$

Besides, the contribution of tensor with two time indecies Y_{4ij4} (similar to Eq.(18)) is non-zero as well. Its components $Y^{44}_{1,2}$ in the limit $\Delta\to\infty$ are the following

$$Y^{44}_1 \to \frac{1}{6}\frac{\pi^2}{\Delta}\ ,\quad Y^{44}_2 \to \frac{33}{16}\frac{\pi^2}{\Delta}\ .$$

Now making use of the definitions of Λ_n (Eq.27) and M^{ak}_n (Eq.28) we find the interaction S^{δ}_{int} contributing the terms of the second order in the parameter $\frac{1}{g}\frac{e}{4\pi}$ to the action. Those terms are quite comparable with the contribution of S^{d}_{int} in the regime interesting for physics when the source intensity e is of the same order as g (the factor 4π is obviously compensated while integrating). The suppression of multipole contributions is expected to come from the geometric factor which emerges while integrating over the azimuthal angle and for the field configurations rather homogeneous in space, approximately it behaves like $\frac{1}{2k+1}$ where k is the multipole type. Fig.2 where the contributions of S^{d}_{int} (two upper curves) and S^{δ}_{int} (two lower curves) are exhibited manifestly demonstrates the role of interactions of deformable PPs (solid curves correspond to the instanton and the dashed ones correspond to the anti-instanton).

3 Developing an approximate scheme of deformations

Now we are well equipped to find out the optimal instanton deformations. As mentioned above the corresponding formulae can be obtained analytically if we neglect the contribution of S^{κ}_{int} Eq.(17) which is distance dependent. Optimizing the quadratic form in the action Eq.(15) we receive the following system of algebraic equations

$$\begin{aligned} &\kappa\ \delta_i\rho+\tilde v^{al}_{ji}\ \delta_j\omega^{al}+\Lambda_i=0\ , \\ &\sigma^{kl}_{ij}\ \delta_j\omega^{al}+\tilde v^{ak}_{ij}\ \delta_j\rho+\bar\eta_{mij}\varepsilon^{abc}v_2\ (2\ \omega^{bm}\delta_j\omega^{ck}-\omega^{bk}\delta_j\omega^{cm})- \\ &-v_2\ \bar\eta_{kij}\varepsilon^{abc}\omega^{bn}\delta_j\omega^{cn}-2v_1\ \varepsilon_{kmn}\varepsilon^{abc}\omega^{bm}\delta_i\omega^{cn}+\frac{4}{3}v_2\ \omega^{ak}\omega^{cn}\delta_i\omega^{cn}+M^{ak}_i=0\ . \end{aligned} \tag{29}$$

In order to resolve it we introduce a new variable

$$y^{jki}=\omega^{aj}\delta_i\omega^{ak}\ .$$

Recalling the orthogonality property of the rotation matrices and invariance of the group measure Eq.(14) we have

$$\begin{aligned} &\sigma_1\ \omega^{al}y^{lki}+\sigma_2\ \delta_{ki}\omega^{al}y_{lnn}-\sigma_2\ \omega^{al}y^{lik}+2v_2\ \varepsilon_{mij}\varepsilon_{dml}\ \omega^{ad}y^{lkj}-v_2\ \varepsilon_{mij}\varepsilon_{dkl}\ \omega^{ad}y^{lmj}- \\ &-v_2\ \varepsilon_{kij}\varepsilon_{dnl}\ \omega^{ad}y^{lnj}-2v_1\ \varepsilon_{kmn}\varepsilon_{dml}\ \omega^{ad}y^{lni}+\frac{4}{3}v_2\ \omega^{ak}y^{nni}+\tilde v^{ak}_{ij}\delta_j\rho+M^{ak}_i=0\ . \end{aligned}$$

Multiplying it by the ω matrix and calculating the products of the antisymmetric tensors we discover the equations

$$
\begin{aligned}
&\sigma_1\; y^{ijk} - \sigma_2\; y_{ikj} + 2v_1\; y^{jik} + v_2\; y^{jki} + v_2\; y^{kij} + \\
&+\delta_{jk}(\sigma_2\; y^{inn} - v_2\; y^{nin} + v_2\; y^{nni}) - v_2\; \delta_{ki}(y^{jnn} + y^{nnj}) + \\
&+\delta_{ji}\left[-v_2\; y^{nkn} + v_2\; y^{knn} + \left(\frac{4}{3}v_2 - 2v_1\right)\; y^{nnk}\right] + v^{ij}_{kn}\; \delta_n\rho + \omega^{ai}M^{aj}_k = 0\ .
\end{aligned} \tag{30}
$$

The solution of this equation system is essentially based on the presence of the convolution of independent variable y_{ijk}. It is clear these equations are not contradictory if the following constraints on three possible convolutions y^{ikk}, y^{kik}, y^{kki} are obeyed

$$
\begin{aligned}
&(\sigma_1 + 2\sigma_2)\; y^{ikk} + (2v_1 - 3v_2)\; y^{kik} + \left(\frac{13}{3}v_2 - 2v_1\right) y^{kki} + \omega^{ai}M^{ak}_k + v^{ik}_{kn}\; \delta_n\rho = 0\ , \\
&(\sigma_2 + 3v_2)\; y^{ikk} - (\sigma_2 + 3v_2)\; y^{kik} + (\sigma_1 - 4v_1 + 4v_2)\; y^{kki} + \omega^{ak}M^{ak}_i + v^{kk}_{in}\; \delta_n\rho = 0\ , \\
&(\sigma_2 + 2v_1 - v_2)\; y^{ikk} + (\sigma_1 - 2v_2)\; y^{kik} + \left(\frac{v_2}{3} - \sigma_2 - 2v_1\right) y^{kki} + \omega^{ak}M^{ai}_k + v^{ki}_{kn}\; \delta_n\rho = 0\ ,
\end{aligned} \tag{31}
$$

For what follows it is practical to analyze a more general situation of the system with arbitrary constraints. The first five terms of Eq.(30) which do not contain the convolutions might be presented using the following auxiliary tensor

$$
s^{jp\beta;kq\gamma} = s_1\; \delta_{jk}\delta_{pq}\delta_{\beta\gamma} + s_2\; \delta_{jk}\delta_{p\gamma}\delta_{\beta q} + s_3\; \delta_{jq}\delta_{pk}\delta_{\beta\gamma} + s_4\; \delta_{jq}\delta_{p\gamma}\delta_{\beta k} + s_5\; \delta_{j\gamma}\delta_{pk}\delta_{\beta q}\ ,
$$

with the components $s_1 = \sigma_1$, $s_2 = -\sigma_2$, $s_3 = 2v_1$, $s_4 = s_5 = v_2$. The inverse tensor has the following form

$$
\tau^{io\alpha;jp\beta} = \tau_1\; \delta_{ij}\delta_{op}\delta_{\alpha\beta} + \tau_2\; \delta_{ij}\delta_{o\beta}\delta_{\alpha p} + \tau_3\; \delta_{ip}\delta_{oj}\delta_{\alpha\beta} + \tau_4\; \delta_{ip}\delta_{o\beta}\delta_{\alpha j} + \tau_5\; \delta_{i\beta}\delta_{oj}\delta_{\alpha\beta} + \tau_6\; \delta_{i\beta}\delta_{op}\delta_{\alpha j}\ ,
$$

and it is valid for this tensor $\tau^{io\alpha;jp\beta}s^{jp\beta;kq\gamma} = \delta_{ik}\delta_{oq}\delta_{\alpha\gamma}$ and its components are

$$
\tau_1 = \frac{11\cdot 199}{D}\ ,\quad \tau_2 = -\frac{25\cdot 31}{D}\ ,\quad \tau_3 = -\frac{35\cdot 37}{D}\ ,
$$

$$
\tau_4 = \tau_5 = \frac{109}{D}\ ,\quad \tau_6 = \frac{7\cdot 23}{D}\ ,\quad D = 13\cdot 83\ \beta\ .
$$

In principle these 'strength' coefficients could be presented in the terms of corresponding combinations of σ_i and v_i. However, they look like too much sophisticated and it is the main reason for showing their overt expressions here for PP in the singular gauge. It is clear that the relations obtained are enough to get the solution of the system Eq.(30). The convolutions are extracted from the constraint equations Eq.(31)

$$
\begin{pmatrix} y^{ikk} \\ y^{kik} \\ y^{kki} \end{pmatrix} = \left\| \begin{matrix} -\dfrac{136}{25\ \beta} & \dfrac{24}{175\ \beta} & \dfrac{8}{5\ \beta} \\ & -\dfrac{24}{7\ \beta} & \\ \dfrac{16}{25\ \beta} & -\dfrac{144}{175\ \beta} & -\dfrac{8}{5\ \beta} \end{matrix} \right\| \begin{pmatrix} \omega^{ai}M^{ak}_k + (v_1 - 2v_2)\delta_i\rho \\ \omega^{ak}M^{ak}_i + 3v_1\delta_i\rho \\ \omega^{ak}M^{ai}_k + (v_1 + 2v_2)\delta_i\rho \end{pmatrix}\ .
$$

When it is used in the equations for $\delta\rho$ we obtain

$$
-\frac{53}{200}\ \beta\ \delta_n\rho - \frac{28}{25}\ \omega^{ak}M^{ak}_n + \frac{19}{25}\ \omega^{an}M^{ak}_k - \frac{2}{5}\ \omega^{ak}M^{an}_k + \Lambda_n = 0\ . \tag{32}
$$

Making use of the tensor τ which is inverse to the tensor s of the system Eq.(30) we find out the solution y and reconstruct the solution $\delta_i\omega^{ak} = \omega^{aj}y^{jki}$. The overt formulae are too cumbersome and are not shown here.

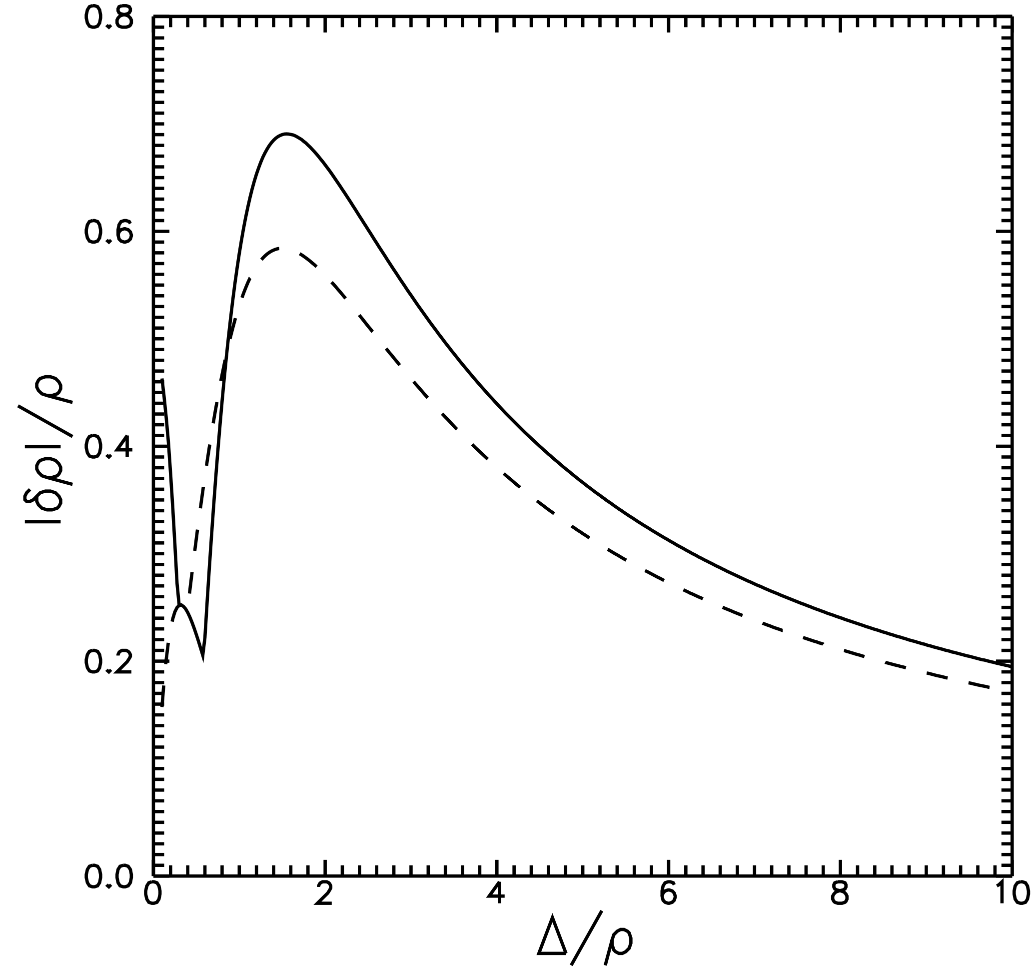

Figure 3: The norm of instanton size deviations $|\delta\rho|$ for version with the parameters mentioned in Fig.1 caption. The dashed line corresponds to the anti-instanton.

4 Analysing numerical results

The most interesting question we are going to answer here is what are the distances where the application of the perturbative approach developed is still valid and reliably justified. Actually, the formal criterion could be grounded on the deformation smallness on the scale of characteristic PP size which corresponds to the distance of order $|y| \simeq 1$ in the dimensionless variables. Fig.3 illustrates the behaviour of the norm of instanton size deviations

$$|\delta\rho| = \max_i |\delta_i \rho| \ , \quad i = 1, 2, 3 \ ,$$

as the function of the path. That is clearly seen the perturbative approach is completely relevant at small distances and distances commensurate with several instanton sizes. Besides, we could also conclude the deviations from asymptotic PP size are strongly dependent on the mutual positions of PP and point-like source in the isotopic space. The example of Fig.3 is one of the most rigorous limitations in $|\delta\rho|$. If one orientates an instanton along the third axis of isotopic space collinearly with the point-like source the deformations get considerably smaller. This configuration in isotopic space is characterized by the prevailing contribution of rotational mode to S^{δ}_{int} Eq.(16) comparing to the dilatational one. However, if one orientates PP along the direction of point-like source vector, the corresponding contribution of dilatational mode becomes prevailing, i.e. it is more advantageous for PP to rotate in the isotopic space being correlated with the point-source. Fig.4 displays the total increase of instanton action $(S - \beta)$ (solid curve) and anti-instanton (dashed curve) as function of the path. The upper solid line shows the contribution of S^{d}_{int} from Eq.(16) in the proximity of the

coordinate origin. Let us notice here that at large distances both the instanton trajectory and the anti-instanton one are running above S^d_{int} which is just the action corresponding to non-deformed instanton. It means the crumpled PP develops the action maximum in this region and, hence, the paths in the functional space might exist and the PP action might be decreasing while going along them.

In order to appoint those we consider more general problem if determine the extremal value of the functional Eq.(15) with nine arbitrary constraints

$$y^{ikk}=a_i\ ,\quad y^{kik}=b_i\ ,\quad y^{kki}=c_i\ .$$

The action Eq.(15) with the Lagrange multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$ might be transformed into the form

$$S\to\hat S=S+\lambda_i(y^{ikk}-a_i)+\mu_i(y^{kik}-b_i)+\nu_i(y^{kki}-c_i)\ . \tag{33}$$

In fact, it is not easy to diagonalize directly the quadratic form for this system. Nevertheless, the useful information on the general construction of this function can be extracted. Trying to do that we rewrite the part related to the rotations in the isotopic space

$$\begin{aligned} &S^y=y^{ip\alpha}\ \sigma^{ip\alpha;jp\beta}\ y^{jp\beta}\ ,\\ &\sigma^{ip\alpha;jp\beta}=\frac{\sigma_1}{2}\ \delta_{ij}\delta_{pq}\delta_{\alpha\beta}-\frac{\sigma_2}{2}\ \delta_{ij}\delta_{p\beta}\delta_{\alpha q}+v_2\ \delta_{iq}\delta_{p\beta}\delta_{\alpha j}+v_1\ \delta_{iq}\delta_{pj}\delta_{\alpha\beta}+\\ &+\frac{\sigma_2}{2}\ \delta_{ij}\delta_{p\alpha}\delta_{q\beta}-v_2\ \delta_{iq}\delta_{p\alpha}\delta_{j\beta}-v_2\ \delta_{i\alpha}\delta_{p\beta}\delta_{jq}-v_2\ \delta_{i\beta}\delta_{p\alpha}\delta_{jq}+\left(\frac{2}{3}v_2-v_1\right)\delta_{ip}\delta_{\alpha\beta}\delta_{jq}\ , \end{aligned} \tag{34}$$

using the multiplet expansion in the form

$$y^{jq\beta}=y^{\{jq\beta\}}+y^{[jq\beta]}+y^{\{j[q\}\beta]}+y^{[j\{q]\beta\}}\ ,$$

where the combinations symmetric in indecies are denoted by the curly brackets and antisymmetric ones are given by the square brackets. Then the terms without convolutions can be written in the diagonal form as

$$\begin{aligned} &S^y=y^{ip\alpha}\Big[r_1\ y^{\{ip\alpha\}}+r_2\ y^{[ip\alpha]}+r_3\ y^{\{i[p\}\alpha]}+r_4\ y^{[i\{p]\alpha\}}+\\ &+\delta_{p\alpha}\left(\frac{\sigma_2}{2}\ y^{ikk}-v_2\ y^{kik}+v_2\ y^{kki}\right)-v_2\ \delta_{i\alpha}y^{kkp}+\delta_{ip}\left(\frac{2}{3}\ v_2-v_1\right)y^{kk\alpha}\Big]\ , \end{aligned} \tag{35}$$

where the eigenvalues are defined as

$$\begin{aligned} r_1&=\frac{\sigma_1}{2}-\frac{\sigma_2}{2}+v_2+v_1=\frac{13}{12}\beta\ ,\quad r_2=\frac{\sigma_1}{2}+\frac{\sigma_2}{2}+v_2-v_1=\frac{1}{8}\beta\ ,\\ r_3&=\frac{\sigma_1}{2}+\frac{\sigma_2}{2}+v_1-v_2=\frac{11}{24}\beta\ ,\quad r_4=\frac{\sigma_1}{2}-\frac{\sigma_2}{2}-v_2-v_1=\frac{1}{4}\beta\ . \end{aligned} \tag{36}$$

As all r_i are positive we find every separate multiplet to be stable and clearly distinguishable because of their contribution to the term of kinetic energy type. The system of equations with arbitrary constraints imposed looks like

$$\begin{aligned} &\kappa\delta_i\rho+v_1y^{kki}+v_2y^{kik}+v_2y^{ikk}+\Lambda_i=0\ , \qquad (37)\\ &\sigma_1y^{ijk}-\sigma_2y_{ikj}+2v_1y^{jik}+v_2y^{jki}+v_2y^{kij}+\delta_{jk}(\sigma_2y^{inn}-v_2y^{nin}+v_2y^{nni})-v_2\delta_{ki}(y^{jnn}+y^{nnj})+\\ &+\delta_{ji}\left[-v_2y^{nkn}+v_2y^{knn}+\left(\frac{4}{3}v_2-2v_1\right)y^{nnk}\right]+\lambda_i\delta_{jk}+\mu_j\delta_{ik}+\nu_k\delta_{ij}+v^{ij}_{kn}\delta_n\rho+\omega^{ai}M^{aj}_k=0\ . \end{aligned}$$

Then the conditions for this system not to be the contradictory one might be taken as the equations for the Lagrange multipliers

$$\begin{aligned} &3\boldsymbol{\lambda}+\boldsymbol{\mu}+\boldsymbol{\nu}+(\sigma_1+2\sigma_2)\boldsymbol{a}+(2v_1-3v_2)\boldsymbol{b}+\left(\frac{13}{3}v_2-2v_1\right)\boldsymbol{c}+\omega^{ai}M^{ak}_k+(v_1-2v_2)\delta\boldsymbol{\rho}=0\ ,\\ &\boldsymbol{\lambda}+3\boldsymbol{\mu}+\boldsymbol{\nu}+(\sigma_2+3v_2)\boldsymbol{a}-(\sigma_2+3v_2)\boldsymbol{b}+(\sigma_1-4v_1+4v_2)\boldsymbol{c}+\omega^{ak}M^{ak}_i+(v_1+2v_2)\delta\boldsymbol{\rho}=0\ ,\\ &\boldsymbol{\lambda}+\boldsymbol{\mu}+3\boldsymbol{\nu}+(\sigma_2+2v_1-v_2)\boldsymbol{a}+(\sigma_1-2v_2)\boldsymbol{b}+\left(\frac{v_2}{3}-\sigma_2-2v_1\right)\boldsymbol{c}+\omega^{ak}M^{ai}_k+3v_1\delta\boldsymbol{\rho}=0\ . \end{aligned}$$

The equation for the size deformation reads

$$\kappa \delta \boldsymbol{\rho} - v_2 \boldsymbol{a} + v_2 \boldsymbol{b} - v_1 \boldsymbol{c} + \boldsymbol{\Lambda} = 0 \ .$$

These equations provide us with $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$ and putting those in Eq.(37) we come (using the inverse tensor τ) to the solution for the system with constraints. At large distances from the charge where the interaction is negligible or in the particular case of the interaction absent (the deformed PP only is considered) the Lagrange multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu}$ and solution y^{ijk} are the functions of constraints $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ only

$$\begin{pmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \boldsymbol{\nu} \end{pmatrix} = \begin{Vmatrix} -\frac{83}{D_1} & -\frac{43}{D_1} & -\frac{49}{3\ D_1} \\ -\frac{43}{D_1} & -\frac{353}{D_1} & \frac{571}{3\ D_1} \\ -\frac{49}{3\ D_1} & \frac{571}{3\ D_1} & -\frac{557}{9\ D_1} \end{Vmatrix} \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \end{pmatrix} ,$$

where $D_1 = \dfrac{5 \cdot 9 \cdot 32}{\beta}$.

$$\begin{aligned} y^{ijk} &+ \delta^{jk}(A_{11}\ a_i + A_{12}\ b_i + A_{13}\ c_i) + \delta^{ik}(A_{21}\ a_j + A_{22}\ b_j + A_{23}\ c_j) + \\ &+ \delta^{ij}(A_{31}\ a_k + A_{32}\ b_k + A_{33}\ c_k) = 0 \ , \end{aligned}$$

moreover, the matrix A is just a projector necessary to excrete from the solution y nine constraints imposed $A_{11} = A_{22} = A_{33} = -\frac{2}{5}$, $A_{12} = A_{13} = A_{21} = A_{23} = A_{31} = A_{32} = \frac{1}{10}$. Then as the result the action Eq.(33) becomes the quadratic form of three constraint vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ i.e.

$$\hat{S} = -\frac{1}{2}(\boldsymbol{\lambda a} + \boldsymbol{\mu b} + \boldsymbol{\nu c}) \ .$$

Two eigenvalues of this form are positive and one is negative. The direction where going along the action is getting smaller is given by the following equation

$$\boldsymbol{a} = \boldsymbol{0} \ , \quad 3 \cdot 571\ \boldsymbol{b} - 557\ \boldsymbol{c} = \boldsymbol{0} \ ,$$

(let us remind once more that these result was obtained considering the instanton in singular gauge, and we omitted their representations while the corresponding components of tensors σ_i, v_i because of their very cumbersome forms). We did not manage to diogonalyze equations (30) manifestly nevertheless imposing arbitrary constaints we demonstrate the part of eigenvalues of quadratic form is negative and PP can manifest itself as a 'sphaleron'. In principle studying the deformed PPs out of the perturbation theory developed here could allow to understand transition of nonperturbative field configurations into the perturbative ones. Usually such a transition is associated with instanton anti-instanton annihilation only and justifies the criticism of simple superposition ansatz in the IL model [6]. From this view point we demonstrate that already in one particle sector there exist the ways in the functional space which inter-relate the solutions from different topological classes. However, it is a well known fact that the solution with nontrivial topology in spite of the loss of action is characterised by large statistical weight.

Dealing with one PP we could study the deformations which leave the solution within the same topological class if the condition of topological charge conservation $\delta N = 0$ is imposed. In the approximation of quadratic deviations we receive the following result

$$\delta N = \frac{1}{4\beta} \int dx\ G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} - 1 = -\frac{3}{10}\ \delta_\mu \rho \delta_\mu \rho - \frac{1}{12}\ \delta_\mu \omega^{ak} \delta_\mu \omega^{ak} \ , \tag{38}$$

where $\widetilde{G}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta}$. It leads to the conclusion that the dipole-like deformations (the expansion Eq.(7) is meant) are forbidden if the toplogical charge is not changed.

As an example of the system with the constraints imposed we consider the substitution with four non-zero components (which is dictated by the particular form of matrix ω)

$$\delta_1\rho = x \ , \quad \delta_1\omega^{13} = \delta_1\omega^{22} = y \ , \quad \delta_1\omega^{31} = z \ .$$

Besides, the contribution of terms S^{κ}_{int} could be analysed in this example. The solution is determined by minimizing the following quadratic form

$$S - \beta = \frac{A}{2}\, x^2 + \frac{B}{2}\, y^2 + \frac{9}{16}\beta\, z^2 + Fyz + Cxy + Dxz + \Lambda_1 x + Ey + Gz \ ,$$

where

$$\begin{aligned}
\frac{A}{2} &= \frac{\kappa}{2} \mp \frac{1}{g}\frac{e}{4\pi}\Sigma_{111} + \left(\frac{e}{4\pi}\right)^2 (2Q_{11} - R_{2211} - R_{3311}) \ , \\
\frac{B}{2} &= \frac{17}{24}\beta \pm \frac{1}{g}\frac{e}{4\pi}\ (2T_{111} - S_{133} - S_{122}) + \left(\frac{e}{4\pi}\right)^2 (2U_{11} - V_{2211} - V_{3311}) \pm \\
&\pm \frac{1}{g}\frac{e}{4\pi}\ 2\ (Z_1 - Z_2 - Z_3 + Z_4) \ , \\
C &= \frac{7}{12}\beta \pm \frac{1}{g}\frac{e}{4\pi}\ 2\ (2W_{111} - 3X_{221} - X_{331}) \pm \frac{1}{g}\frac{e}{4\pi}\ 4\ (\Xi_1 - \Xi_2 - \Xi_3 + \Xi_4) \ , \\
D &= -\frac{7}{24}\beta \mp \frac{1}{g}\frac{e}{4\pi}\Psi_{111} \pm \frac{1}{g}\frac{e}{4\pi}\ 2\ (\Xi_1 - \Xi_2 - \Xi_3 + \Xi_4) \pm \\
&\pm \frac{1}{g}\frac{e}{4\pi}\ 2\ (3\Theta_1^{44} + \Theta_2^{44} + 21\Theta_1 + 12\Theta_2 + \Theta_3) \ , \\
F &= \frac{13}{24}\beta \pm \frac{1}{g}\frac{e}{4\pi}\ 2\ (3\Phi_1^{44} + \Phi_2^{44} + 21\Phi_1 + 12\Phi_2 + \Phi_3) \\
E &= M_1^{13} + M_1^{22} \ , \qquad G = M_1^{31} \ ,
\end{aligned}$$

here the upper sign corresponds to the instanton. The instanton trajectory is depicted by the dotted line in Fig.4.

The topological charge change for the trajectories under consideration is plotted in Fig.5 and allows one to conclude that the interaction does not take the solution out of the topological class with the charge $|N| = 1$. The quantity defined by Eq.(38) characterizes the deformation extent of aproximate instanton solution only, also one should pay interest to topological charge of superposition solution on the whole.

Another interesting remark as to the possible constraint conditions comes from their optimization, i.e. from the analysis of nine constraints obtained for the action extremum available. It turns out the initial trajectories of Eq. (31) are just the case and the Lagrange multipliers for this extremal action are trivial ($\boldsymbol{\lambda} = \boldsymbol{\mu} = \boldsymbol{\nu} = \mathbf{0}$).

5 Analyzing the IL model

The results obtained above allow one to conclude the perturbative approach developed is applicable to effectively explore the external field impact on PP no matter how large (or small) the distances are. One should expect the substantial suppression of instantons at the distances of average instanton size order in IL but the Coulomb-like interaction extends to the distances much larger, of several characteristic instanton sizes.

Apparently, the configurations of crumpled instantons may come about very essential for the IL model. The entropy of crumpled pseudoparticles and their contribution to the functional integral are of the same order as for the habitual instantons. (The entropy of a field configuration is implied

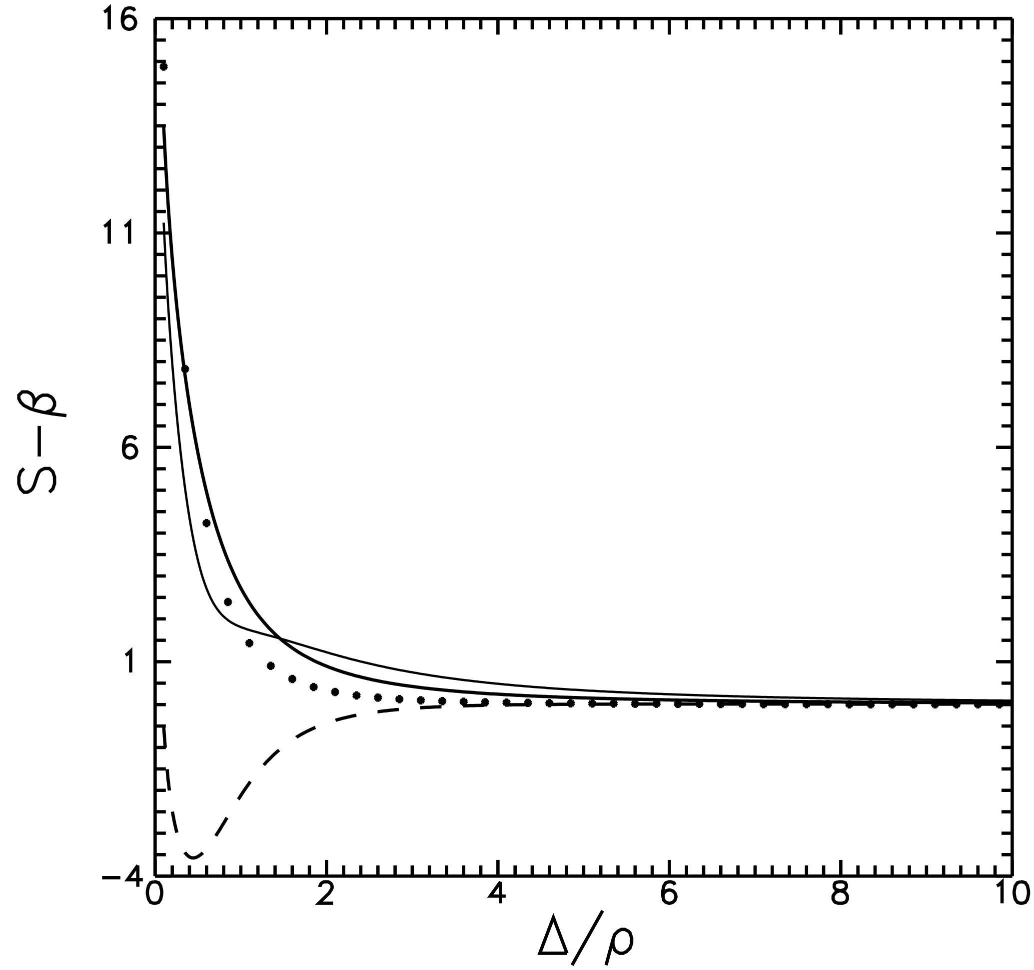

Figure 4: The increase of action $(S-\beta)$ for the crumpled instanton and anti-instanton (dashed line). The parameters of the rotation matrix and the path are mentioned in the caption of Fig.1. Two solid lines correspond to the instanton and the upper one (around the coordinate origin) shows S^d_{int}. The dotted line shows the instanton trajectory with the constraints imposed.

as the *log* of the volume in functional space occupied by similar configurations.) An importance of searching the similar trajectories which have the large entropy (even losing in the action) at calculating the functional integral is well known (see, for example, [13]). In fact, the condition $\delta S = 0$ for searching the optimal crumpled configurations operates with the interaction contributions coming from exponential factor only and, as can be shown, is a valley equation in quasi-clasical approximation. In order to pretend to higher precision and to take into account so-called quantum interactions one should deal with the valley method (see for example [14] where the similar practical method is given). However, the discussed example of dipole-like deformations Eq.(7) demonstrates obviously how sophisticated this configuration can happen to be even in this comparatively simple case.

Another remark worth mentioning here concerns the behaviour of the pseudo-particle deformations at large distances. Studying the asymptotic behaviour of interaction terms one easily finds out their Coulomb-like character, i.e. even in far distant area the pseudo-particle field may be noticeably polarized. In principle, if it turns out possible in some way to detach the long-range component on the background of inter-instanton interactions (for example, for dilute gas of very cooled (anti-)instanton configurations) the chance may appear to measure the coupling constant g on the lattice directly if an external field of intensity e (which is controlled) is applied to its boundary. The dilatational and rotational deformations may contribute to the observables only as the corresponding averages $\langle \delta_n \rho \rangle$, $\langle \delta \omega^{ab}_i \rangle$ which are defined by two independent parameters e^2 and $\frac{e}{g}$. It looks feasible to construct such

a linear combination of those two averages which depends on the intensity e only and certainly should be under control during the measuring process whereas another could provide us with the magnitude of g. The great experience collected in lattice QCD to work with the boundary conditions imposed [15] gives us the hope to succeed on this way applying the cooling technics [16] which isolates the instanton configurations successfully and incidently they look quite deformed [17].

5.1 Crumpling instantons in the IL model

Here we are trying to suggest a fairly simple approach of taking into account the crumpled pseudoparticle configurations in the functional integral. If we pretend to make this estimate calculating precisely the exponential factor of generating functional only then their contribution could be found in quasi-classical approximation just in the same way as it is done for the undeformed pseudo-particles (see the designations in [12])

$$Z = \sum_N \frac{1}{N!} \prod_{i=1}^{N} \int \frac{d\gamma_i}{V\ \rho_i^5}\ C_{N_c}\ \widetilde{\beta}^{2N_c}\ e^{-S}\ , \qquad d\gamma_i = dz_i\ d\rho_i\ d\omega_i. \tag{39}$$

As supposed in the IL model this integral is saturated by the configurations of characteristic size $\bar{\rho}$. In the approach under consideration S means an action including the deformations in the above formula. Thus, adding the contibution of the crumpled pseudo-particles of the similar characteristic scale we are able to work within the same precision studying the IL reaction on the external impact. The variational Ritz method as defined by Eq.(6) looks to be pretty hard and incomplete from the viewpoint of the integration in the functional space. We have permitted the velocities and higher coefficients of multipole expansion only to vary there but not the major expansion term that is the average instanton size. In view of the functional integration expanding the average pseudo-particle size varied seems more natural. Then we have for this particular expansion the following expression

$$R_{in}(x,z) = \rho(z) + \frac{\partial \rho(z)}{\partial z_\mu}\ y_\mu + \frac{\partial^2 \rho(z)}{\partial z_\mu \partial z_\nu}\ \frac{y_\mu\ y_\nu}{2} + \dots\ , \qquad |y| \leq L\ . \tag{40}$$

and the similar one for Ω. The generating functional becomes now

$$Z = \int D[\rho(z)]\ D[\omega(z)] \sum_N \frac{1}{N!} \prod_{i=1}^{N} \int \frac{dz_i\ d\omega_i}{V}\ C_{N_c}\ \widetilde{\beta}^{2N_c}\ e^{-S}\ . \tag{41}$$

Remembering that the IL approach, as formulated, takes into account interactions between the pseudo-particles we should expect in Eq.(9) the pseudo-particle as the field B, too. In order to simplify the calculations let us suppose for the moment the (anti-)instantons undergo the insignificant changes of size because of an impact of external source on IL, i.e. $\rho(z) = \bar{\rho} + \delta\rho(z)$ and colour instanton orientation gets $\omega_i(z) = \omega_i + \delta\omega(z)$ and for anti-instanton it is $\bar{\omega}_i(z) = \bar{\omega}_i + \delta\bar{\omega}(z)$. The first terms of ω correspond to the anticipated fixed directions in colour space. The field strength of single instanton looks like

$$G^a_{\mu\nu}(A) = -\frac{4}{g}\ \omega^{ak}(z) M_{\mu\alpha} M_{\nu\beta}\ \bar{\eta}_{k\alpha\beta}\ \frac{\rho^2(z)}{(y^2+\rho^2(z))^2}\ ,$$

with ω changed for $\bar{\omega}$ for the anti-instanton. When we have the instanton-instanton 'molecule' the mixed component of field strength $G(A,B)$ reads

$$G^a_{\mu\nu}(A,B) = \frac{4}{g}\ \varepsilon^{abc}\omega^{bn}(z_1)\omega^{ck}(z_2)\ (\bar{\eta}_{n\mu\gamma}\bar{\eta}_{k\nu\alpha} - \bar{\eta}_{n\nu\gamma}\bar{\eta}_{k\mu\alpha})\ \frac{y_{1\alpha} y_{2\gamma}}{y_1^2 y_2^2}\ \frac{\rho^2(z_1)}{(y_1^2+\rho^2(z_1))}\frac{\rho^2(z_2)}{(y_2^2+\rho^2(z_2))}$$

where $y_i = x - z_i$. It is clear all the combinations of colour matrices for the instanton-instanton interaction owing to the property Eq.(14) give the unit matrix if we limit ourselves with the contact

type of interaction only. This approximation seems well justified since the integrals of our interest are quickly decreasing for the far distant pseudo-particles $|z_1 - z_2| \gg \bar{\rho}$. For the instanton-anti-instanton molecule such a component is given by

$$G^a_{\mu\nu}(A,B) = \frac{4}{g}\ \varepsilon^{abc}\bar{\omega}^{bn}(z_1)\omega^{ck}(z_2)\ (\eta_{n\mu\gamma}\bar{\eta}_{k\nu\alpha} - \bar{\eta}_{k\mu\alpha}\eta_{n\mu\gamma})\ \frac{y_{1\alpha}y_{2\gamma}}{y_1^2 y_2^2}\ \frac{\rho^2(z_1)}{(y_1^2+\rho^2(z_1))}\frac{\rho^2(z_2)}{(y_2^2+\rho^2(z_2))}\ .$$

The properties of the 't Hooft symbol make it possible to demonstrate that the product of matrices ω and $\bar{\omega}$ gives the unit one in this case as well. Then the averaged integral for ineracting pseudo-particles may be written in the following form (see Ref. [12] for designations)

$$\begin{aligned} \langle S(A,B)\rangle &= \int \frac{dz_1}{V} \int \frac{dz_2}{V}\ \bar{U}_{int}(A,B)\ , \\ \bar{U}_{int}(A,B) &= \frac{8\pi^2}{g^2}\ \frac{N_c}{N_c^2-1}\int dx\ \frac{(7\ y_1^2y_2^2 - (y_1y_2)^2)\ \rho^2(z_1)\rho^2(z_2)}{y_1^4(y_1^2+\rho^2(z_1))^2\ y_2^4(y_2^2+\rho^2(z_2))^2}\ . \end{aligned} \tag{42}$$

And integrating over the positions of single detached pseudo-particle one may receive an estimate in the form of contact term

$$\int dz_1\ \bar{U}_{int}(A,B) \simeq \frac{8\pi^2}{g^2}\ \xi^2\ \rho^2(z_2)\rho^2(z_2) \simeq \frac{8\pi^2}{g^2}\ \xi^2\ \bar{\rho}^2\rho^2(z_2)\ ,$$

where $\xi^2 = \frac{27\pi^2}{4}\frac{N_c}{N_c^2-1}$. (Obviously, the interaction integral is independent of the colour orientation because of the IL isotropy in colour space.) Besides, we should take into account the term of kinetic energy type Eq.(11) for each pseudo-particle. As the result, it has been shown in [7], the generating functional (41) for such a sort of crumpled pseudo-particles takes the form of the effective Lagrangian for the scalar field $\rho(z)$ with the mass gap $M^2 = \frac{2\ (11N_c - 2N_f - 12)}{3\ \kappa\ \bar{\rho}^2}$. The dependence on the rotational component ($\delta\omega$, $\delta\bar{\omega}$) occurs to be trivial. The estimate for kinetic coefficient $\kappa \sim 1.5\beta$—6β has been obtained before and depends on the ansatz for the saturating configuration. For example, for pure gluodynamics with the parameters $N_c = 3$, $\beta \simeq 17.5$, $\bar{\rho}\Lambda \simeq 0.37$ at $\kappa = 4\beta$ it is $M \simeq 1.21\Lambda$. In this case we have the soft mass scale with characteristic for IL Λ –280 MeV. The result $\kappa = 0.9\beta$ we have received in this paper corresponds to essentially harder mass scale, above 1 GeV. The IL excitation discussed in its quantum numbers could be identified as a glueball.

5.2 Average source energy in IL

In this paper we are studying the way in which an external field acts upon the pseudo-particle. If we don't fix the features of field $B^a_\mu(x)$ in advance the feedback of IL could be described self-consistently. Then the multipole expansion we used allows us to factorize the problem in a sense and construct the closed equation for the Fourier component of the field $B^a_\mu(p)$. That equation looks rather promising to calculate the Debay screening radius that is one of the most important physical parameters characterizing IL. However, even the description we have already elaborated for the (anti-)instanton behaviour in external field is pretty rich to provide us with interesting information on the IL properties and possibility to explore the various observables. We mention here one result only calculating an average energy of Euclidean source inserted into IL. As a relevant saturating configuration in the IL model one takes the superposition of (anti-)instanton fields with the source field B^a_μadded as

$$A^a_\mu(x) = B^a_\mu(x) + \sum_{i=1}^{N} A^a_\mu(x;\gamma_i)\ , \tag{43}$$

where $\gamma_i = (\rho_i, z_i, \omega_i)$ denotes the parameters describing the i-th instanton. It is seen in Fig. 4 that at the distances larger than $2\Delta/\rho$ the IL density becomes practically equal to its asymptotic value

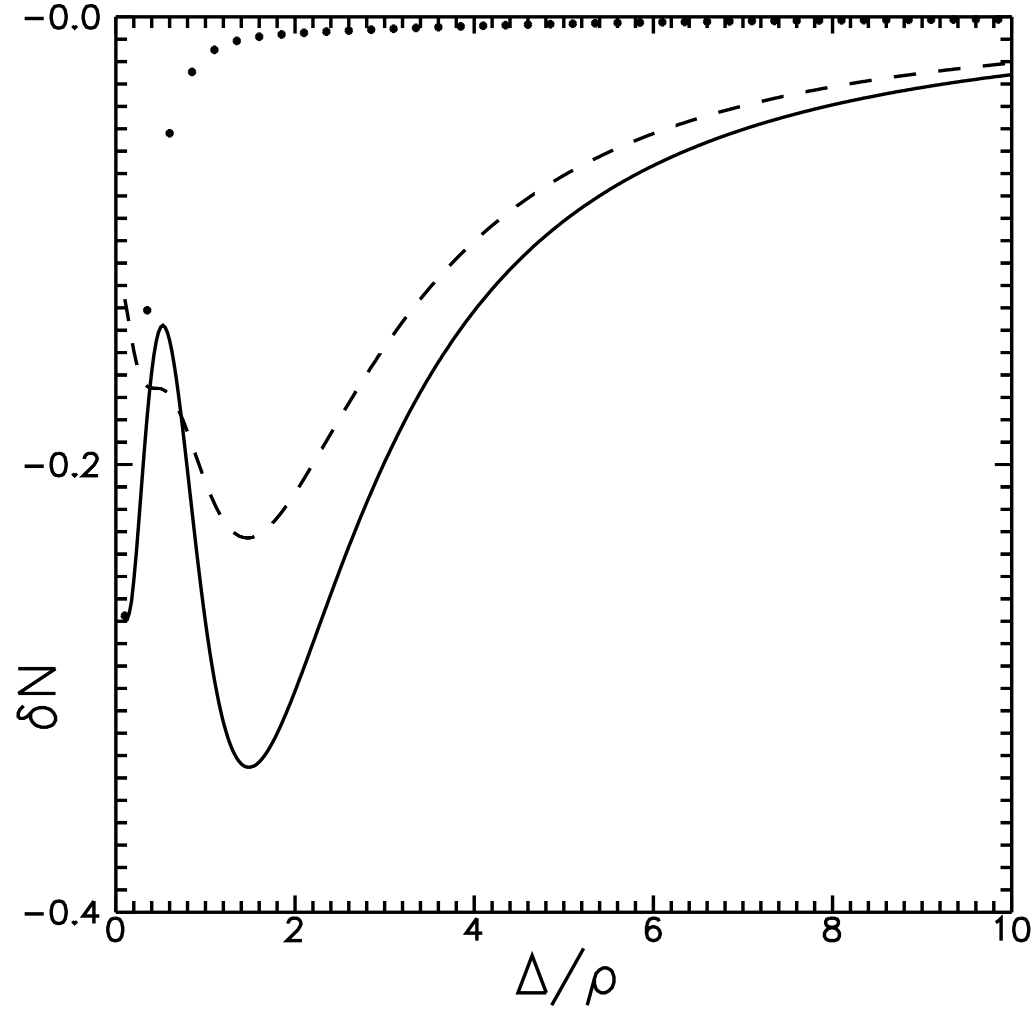

Figure 5: Behaviour of the topological charge δN for the crumpled instanton and anti-instanton (dashed line). The parameters of rotation matrix and path are the same as in Fig.1. The dotted line corresponds the trajectory for instanton with the constraints imposed, see the text.

$n(\Delta) \sim n_0 e^{\beta - S} \simeq n_0$ because the action of each pseudo-particle there approximately coincides with β, $S - \beta \geq S^d_{int}$. The quantity here we are interested in should be given by averaging S over the pseudo-particle positions and their colour orientations (taking all the pseudo-particles of the same size for simplicity) as

$$\begin{aligned} \langle S \rangle &= \prod_{i=1}^{N} \int \frac{dz_i}{V} \int d\omega_i \;\; S = \\ &= \frac{e^2}{4\pi}\frac{1}{a}\; X_4 + N\;\beta + N \int \frac{d\boldsymbol{\Delta}}{L^3} \left(\frac{e}{4\pi}\right)^2 \left(J + \frac{K_{ii}}{3}\right) \;, \end{aligned} \tag{44}$$

where L is a formal upper integration limit, $V = L^3 X_4$ defines the IL volume, X_4 is an upper bound of the 'time' integration, N denotes the total number of pseudo-particles and a is a source size value (on strong interaction scale, of course). Taking the asymptotic values of J and K functions and returning to the dimensional variables for the moment one instanton contribution to the average action can be written down in the following form

$$\langle S \rangle \simeq \frac{e^2}{4\pi}\frac{1}{a}\; X_4 + \frac{N}{V}\;\beta\; L^3 X_4 + \frac{N}{V}\frac{6\pi^3}{\beta}\frac{e^2}{g^2}\;\bar{\rho}^2\; L\; X_4 \;,$$

where $\bar{\rho}$ is the mean size of (anti-)instanton in IL. The result is given in the form with the common factor X_4 extracted because our concern now is the (anti-)instanton behaviour in the source field

background where the field is steady and the solution possesses the scaling property at any x_4 slice. In the limit $N, V \to \infty$ and at the IL density $n = N/V$ fixed this result becomes

$$\langle S \rangle \simeq E\ X_4\ , \quad E = \frac{e^2}{4\pi}\frac{1}{a} + n\ \beta\ L^3 + n\ \frac{6\pi^3}{\beta}\frac{e^2}{g^2}\ \bar{\rho}^2\ L\ .$$

Formally, the last term of E looks like a small correction to the gluon condensate (the second term). However, this contribution linearly increasing with L and proportional to e^2 has a different physical meaning of an additional contribution to the source self-energy

$$E \simeq \sigma\ L\ , \quad \sigma = n\ \frac{6\pi^3}{\beta}\frac{e^2}{g^2}\ \bar{\rho}^2\ .$$

The tension value σ for the IL characteristic parameters $\frac{\bar{\rho}}{\bar{R}} \simeq \frac{1}{3}$ where where $\bar{R}$ is the mean separation of pseudo-particles, $n = \bar{R}^{-4}$, $\beta \simeq 12$, $\bar{\rho} \simeq 1\ GeV^{-1}$ [3],[4] comes about $\sigma \simeq 0.6$ GeV/fm (if one takes for the brief estimate for the source intensity $e \simeq g$). This result looks fully relevant in view of the qualitative character of estimates which IL is able to provide. Moreover, such a value is in reasonable agreement with the estimates extracted from the potential models for heavy quarkonia, for example. If one intends to explore the magnitudes like $\langle S_{int}\ e^{-S_{int}} \rangle$ (which could model an effect of suppressing the pseudo-particle contribution in the source vicinity) in numerical calculations it becomes evident the linearly increasing behaviour starts to form at $\Delta/\bar{\rho} \sim 3$—4. An interesting phenomenon of energy increase with a distance increasing which we discovered above looks promising for the applications in IL. Indeed, this result teaches the source mass (we should treat an additional contribution found out just in this way) is unboundedly increasing (if the screening mechanism is not included) what could be interpreted as preventing the possibility to immerse a bare colour charge into IL. Our estimate of asymptotic energy of the Euclidean source put into IL shows the major contribution to the generating functional in quasi- classical approximation while all coupling constants are fixed at the scale of average instanton size $\bar{\rho}$.

Further we are going to exploit this interesting feature of IL for analysis of the colour dipole behaviour dealing with the point-like sources $e\delta^{a3}$, $-e\delta^{a3}$ which are oriented along the third axis of isotopic space and placed in the points $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$, respectively. Then the strength tensor component of our interest looks as follows

$$G^a_{4i}(A,B) = 2\ \frac{e}{4\pi}\ \varepsilon^{a3c}\ \Omega^{ck}\ \bar{\eta}_{ki\alpha}\ \frac{y_\alpha}{y^2}\ \frac{\rho^2}{(y^2+\rho^2)}\ \left(\frac{1}{|\boldsymbol{y}+\boldsymbol{\Delta}_1|} - \frac{1}{|\boldsymbol{y}+\boldsymbol{\Delta}_2|}\right)\ ,$$

where $\boldsymbol{\Delta}_{1,2} = \boldsymbol{z} - \boldsymbol{z}_{1,2}$. The contribution to the average energy E at large distances which is additional to the Coulomb one is defined by the integral similar to that for the configuration with single source (the same coefficient but distinguishing average over the instanon positions), i.e.

$$I_d = \int d\boldsymbol{\Delta}_1 \left(\frac{1}{\boldsymbol{\Delta}_1^2} - 2\ \frac{1}{|\boldsymbol{\Delta}_1||\boldsymbol{\Delta}_2|} + \frac{1}{\boldsymbol{\Delta}_2^2}\right)\ .$$

When the distance between the sources $l = |\boldsymbol{\Delta}_1 - \boldsymbol{\Delta}_2|$ is going to zero the field disappears and the final result should be zero. The integral may be dependent on two parameters L and l only. The dimensional analysis reveals the integral is a linear function of both parameters but the l-dependence only obeys the requirement of integral petering out at $l \to 0$. In order to determine the coefficient we find easily

$$\frac{I_d}{4\pi} = L - 2\left(L - \frac{l}{2}\right) + L = l\ ,$$

(here the contributions of three integrals are shown separately), where $l = |\boldsymbol{\Delta}_1 - \boldsymbol{\Delta}_2|$ is the distance between charges. And finally we have for the additional contribution to the dipole average energy while in IL

$$E \sim \sigma l\ .$$

The non-singlet in colour states are strongly suppressed in IL by the factor of order of the screening radius $\sim \sigma\ R_D$.

Finally, we would like to make one general remark which is not directly related to IL but seems to be of heuristical importance. We encounter the crumpled configurations of topological nontrivial solutions (pseudo-particles) not only while studying the impact of external fields on them or analyzing their interactions. We discover such configurations, for example, studying the simplest kinks in the problem of double well potential $V(x) = -\lambda(x^2 - \eta^2)^2$ (we are dealing here with an analysis in the Euclidean space). Let us consider the kink at finite interval $(-T, T)$ where $T \gg \omega^{-1}$ if the latter is a characteristic kink size ($\omega^2 = 8\lambda\eta^2$). The approximate solution is given by $x_k(t) \simeq x_{as}(t) = \eta\ \mathrm{th}\frac{\omega t}{2}$ and at the turning points $x_k(T) = x_k(-T) \simeq \eta$ the particle velocity disappears $\dot{x}_k(\pm T) = 0$. The trajectory choice is dictated by the requirement that a particle should pass through the coordinate origin at zero time moment (the coordinate of kink center $t_c = 0$). Apart from this (unique) symmetric kink the trajectories with the biased center which crosses the axis $x = 0$ not only at zero time moment are of special interest. In the vicinity of kink center such solutions are approximately described by $x_c(t) = \eta\ \mathrm{th}\frac{\omega(t - t_c)}{2}$. Then, in order to construct these solutions we consider the time interval larger than the initial one $(-T_2, T_2)$ where $T_2 > T$. There is the trajectory with the kink center located at the coordinate origin which we designate as $\acute{x}_k(t)$, i.e. $\acute{x}_k(t) \simeq x_{as}(t)$, and this particle starts and finishes again with the zero velocity $\dot{x}_k'(\pm T_2) = 0$. Calculating the energy of this particle we find the magnitude which is slightly larger than for the trajectory $x_k(t)$ and the particle climbing the saddle point reaches the points higher than the turning points $x_k(\pm T)$ of the initial trajectory. Now, let us shift the time interval origin for the trajectory $\acute{x}_k(t)$. Thus, we receive the solution $x_c(t) = \acute{x}_k(t + T - T_2)$ with the baised center. For this configuration the particle starts to fall down at the time moment $-T$ to the well with the zero initial velocity and crosses over the axis $x = 0$ at the time moment $T_2 - T$. Then climbing the top again it does not stop at the time moment T and continues to move with a certain velocity. Hence, we need to install a hard barrier to reflect it compelling to move in the opposite direction. Strictly speaking such trajectories are not the equation solutions for the initial potential. The trajectory of particle finishing 'properly' could be constructed by a similar way, however, in distinction from the kink $x_k(t)$ this trajectory $x_c(t)$ is asymmetric. Its right hand wing does not copy absolutely the left hand one. Truely, the difference is quite inconsiderable (exponentially small) if $t_c \ll T$ but becomes very essential when the shift of kink center is of the same order as the interval magnitude T. Generally, the particle could have non-zero velocity both at a start and at a finish. If we remember now that calculating the functional integral we are limited by the special requirement for trajectories to deal with only those which have both ends fixed. Our discussion above teaches it could be satisfied by breaking the equation of motion.

Thus, we are forced to examine the inconventional kinks which are just the particular set of trajectories with the measure in the functional space proportional to the magnitude of interval $\int dt_c = 2\ T$. The kink-anti-kink configurations form a much more sophisticated set of trajectories (with complicated interaction between fragments) which are transformed into standard oscillator (perturbative) trajectories, with oscillation frequency $\Omega^2 = 4\ \lambda\ \eta^2$, while approaching the well bottom. Such trajectory should not be identified with the kink-anti-kink configurations. The instructive message of this example, at least, in the context of previous study is that the exact definition of a kink is of less importance compared to the necessity of taking into account properly the contribution of adequate trajectories in the functional integral. We mean the trajectories of considerable power in the functional space, for example, integrating over their deformations if those contribute significantly to an effective action.

Conclusion

Since the instanton discovery a suggestive way to access an information on the nonperturbative QCD dynamics is to investigate the role of this coherent gluon fields as major objects populating the QCD vacuum. The development of the IL model allowed to obtain the estimates, the bulk instanton properties in vacuum and the lattice simulations have later supported these. Hovewer, the study of instanton behaviour in the external fields, as was understood, is of great importance for resolving the principle QCD problems but the results here are still rather hazy. It was a main motivation to have in this paper as a focal point analysis of the PP behaviour in the field of point-like source. In order to deal with such a task we developed the perturbation approach based on the variation of the instanton parameters. We have formulated the variational problem of searching the optimal PP deformations which is algebraically resolved by the Ritz method within the multipole expansion. The regions of parameter changes interesting for the applications in the IL model were especially analyzed in detail. We demonstrate the solution of algebraic problem might be essentially simplified and the overt formula for the deformation fields in dipole approximation might be obtained if one neglects the contribution to the term of kinetic energy type S^κ which depends on the distance to the perturbation source. The coefficients of the quadratic form in this term of kinetic energy type S^κ are related to PP itself and charaterize, in a sense, its 'pliablity' for changing the size and orientation in the 'isotopic' space. The terms responsible for the interactions between undeformed PP and PP with unchanged parameters with fixed point-like Euclidean source of colour field were calculated. In addition, it was realized the dipole-like contributions to S^d_{int} which are proportional to e/g occur noticeably less in the parameter region characteristic for the IL model at smaller distances than the terms of e^2-order. The effective and rather simple method to take into account the 'crumpled' configurations at calculating approximately the functional integral. The description of instanton behaviour in the field of point-like Euclidean colour source developed in the paper clearly demonstrates that the valley topography in the functional space might be so sophisticated that makes it impossible to investigate the problem in any general formulation. Finally, in the framework of superposition ansatz the estimate of average energy of non-abelian dipole in the IL medium has been found out. This energy for the dipole in colour singlet state escalates linearly with the separation increasing for the point-like sources unscreened. The corresponding coefficient of this dependence develops the magnitude similar numerically to one as inferred from the lattice OCD calculations.

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Appendix

Here we present the integrable terms. We show only the components of the corresponding tensors which are mentioned in the paper The interacting terms linear in $\delta\rho$ are the following

$$A_1 = \frac{4\,\pi^2}{\Delta^2}\left[\frac{8}{3}\,D^5 - \frac{8}{3}\,\Delta^5 - \frac{20}{3}\,D^3 + \frac{9}{2}\,D - \frac{3}{D} + \frac{5}{2\Delta}\,\ln(\Delta + D)\right]\,,$$

$$A_2 = \frac{4\,\pi^2}{\Delta^2}\left[4\,\Delta^5 - 4\,D^5 + 10\,D^3 - \frac{7}{2}\,D + \frac{3}{D} + \frac{2}{D^3} - \frac{15}{2\Delta}\,\ln(\Delta + D)\right]\,,$$

where $\hat{\Delta}_i = \frac{\Delta_i}{\Delta}$ is a unit tensor. The term jA of action Eq.(1) contributes to A_2. The interacting terms linear in $\delta\Omega$ take the form

$$D_1 = \frac{2\,\pi^2}{\Delta^2}\left[\frac{8}{3}\,\Delta^5 + \Delta^3 - \frac{8}{3}\,D^5 + \frac{17}{3}\,D^3 - \frac{7}{2}\,D + \frac{1}{2\Delta}\,\ln(\Delta + D)\right]\,,$$

$$D_2 = \frac{2\,\pi^2}{\Delta^2}\left[8\,D^5 - 8\,\Delta^5 + 5\,\Delta^3 - 25\,D^3 + \frac{53}{2}\,D - 8\,\frac{1}{D} - \frac{3}{2\Delta}\,\ln(\Delta + D)\right]\,;$$

The term $j\ A$ of action Eq.(1) contributes to D_2 as

$$E_1 = \frac{4\ \pi^2}{\Delta^2}\left[\frac{2}{9}\ D^5 - \frac{2}{9}\ \Delta^5 - \frac{\Delta^3}{2} - \frac{D^3}{18} - \frac{D}{12} - \frac{1}{12\Delta}\ \ln(\Delta + D)\right]\ ,$$
$$E_2 = \frac{4\ \pi^2}{\Delta^2}\left[\frac{4}{3}\ \Delta^5 + \frac{\Delta^3}{2} - \frac{4}{3}\ D^5 + \frac{17}{6}\ D^3 - \frac{7}{4}\ D + \frac{1}{4\Delta}\ \ln(\Delta + D)\right]\ ;$$

$$F_1 = \frac{16\ \pi^2}{\Delta^2}\left[\frac{\Delta^5}{9} - \frac{D^5}{9} + \frac{5}{18}\ D^3 - \frac{D}{12} - \frac{1}{12\Delta}\ \ln(\Delta + D)\right]\ ,$$
$$F_2 = \frac{16\ \pi^2}{\Delta^2}\left[\frac{2}{3}\ D^5 - \frac{2}{3}\ \Delta^5 - \frac{5}{3}\ D^3 + \frac{5}{4}\ D - \frac{1}{2D} + \frac{\Delta}{4}\ \ln(\Delta + D)\right]\ .$$

$$H_1 = \frac{4\pi^2}{\Delta^4}\left[-\frac{\Delta^7}{9} - \frac{\Delta^5}{6} + \frac{D^7}{9} - \frac{2}{9}D^5 + \frac{5}{72}D^3 - \frac{D}{48} + \left(\frac{\Delta}{12} + \frac{1}{16\Delta}\right)\ln(\Delta + D)\right]\ ,$$
$$H_2 = \frac{4\pi^2}{\Delta^4}\left[-\frac{\Delta^7}{3} - \frac{\Delta^5}{6} + \frac{D^7}{3} - D^5 + \frac{75}{72}D^3 - \frac{D}{16} - \left(\frac{\Delta}{4} + \frac{5}{16\Delta}\right)\ln(\Delta + D)\right]\ ,$$
$$H_3 = \frac{4\pi^2}{\Delta^4}\left[\Delta^7 + \frac{\Delta^5}{3} - D^7 + \frac{19}{6}D^5 - \frac{85}{24}D^3 + \frac{27}{16}D - \frac{5}{16\Delta}\ln(\Delta + D)\right]\ ,$$
$$H_4 = \frac{4\pi^2}{\Delta^4}\left[\Delta^7 - \frac{\Delta^5}{3} - D^7 + \frac{23}{6}D^5 - \frac{125}{4}D^3 + \frac{35}{16}D - \frac{2}{D} + \frac{35}{16\Delta}\ln(\Delta + D)\right]\ .$$

$$Y_1^{44} = 8\ \pi^2\left[-\frac{\Delta^5}{15} - \frac{\Delta^3}{6} - \frac{\Delta}{8} + \frac{D^5}{15}\right]\ ,$$
$$Y_2^{44} = \frac{8\ \pi^2}{\Delta^2}\left[\Delta^7 + \Delta^5 + \frac{\Delta^3}{8} - D^7 + \frac{5}{2}\ D^5 - 2D^3 + \frac{D}{2}\right]\ .$$

$$\begin{aligned}
Y_1 &= \frac{16\pi^2}{\Delta^4}\left[\frac{1}{75}\left(D^9 - \Delta^9\right) - \frac{\Delta^7}{24} - \frac{\Delta^5}{24} - \frac{11}{600}D^7 + \frac{D^5}{1200} + \frac{D^3}{960} + \frac{D}{640} + \frac{21}{13440\Delta}\ln(\Delta + D)\right]\ ,\\
Y_2 &= \frac{16\pi^2}{\Delta^4}\left[\frac{1}{9}\left(\Delta^9 - D^9\right) + \frac{\Delta^7}{8} + \frac{\Delta^5}{48} + \frac{13}{40}D^7 - \frac{89}{240}D^5 + \frac{31}{192}D^3 - \frac{D}{128} - \frac{21}{26880\Delta}\ln(\Delta + D)\right]\ ,\\
Y_3 &= \frac{16\pi^2}{\Delta^4}\left[\frac{6}{5}\left(D^9 - \Delta^9\right) - \frac{3}{8}\Delta^7 + \frac{\Delta^5}{24} + \frac{201}{40}D^7 + \frac{1943}{240}D^5 + \frac{1177}{192}D^3 + \right.\\
&+ \left.\frac{263}{128}D - \frac{1}{4D} + \frac{21}{384\Delta}\ln(\Delta + D)\right]\ .
\end{aligned}$$

The components of the tensors quadratic in $\delta\rho$ are the following

$$\begin{aligned}
\Sigma_1 &= \frac{4\pi^2}{\Delta^3}\left[\frac{52}{75}\left(D^9 - \Delta^9\right) - \frac{2}{3}\Delta^7 - \frac{184}{75}D^7 + \frac{469}{150}D^5 - \frac{32}{15}D^3 - \right.\\
&- \left.\frac{409}{120}D - \frac{17}{3D} + \frac{2}{3D^3} + \left(\frac{367}{40\Delta} + \frac{9}{4}\Delta\right)\ln(\Delta + D)\right]\ ,\\
\Sigma_2 &= \frac{4\pi^2}{\Delta^3}\left[\frac{52}{75}\left(D^9 - \Delta^9\right) - 2\Delta^7 - \frac{28}{25}D^7 - \frac{77}{50}D^5 + \frac{47}{10}D^3 - \right.\\
&- \left.\frac{709}{120}D - \frac{7}{D} + \frac{1}{D^3} + \left(\frac{367}{40\Delta} + \frac{\Delta}{12}\right)\ln(\Delta + D)\right]\ ,\\
\Sigma_3 &= \frac{4\pi^2}{\Delta^3}\left[\frac{52}{75}\left(D^9 - \Delta^9\right) - \frac{2}{3}\Delta^7 - \frac{184}{75}D^7 + \frac{469}{150}D^5 - \frac{49}{30}D^3 - \right.
\end{aligned}$$

$$- \frac{829}{120}D - \frac{2}{D} + \left(\frac{367}{40\Delta} + \frac{49}{12}\Delta\right)\ln(\Delta + D)\Big] ,$$

$$\Sigma_4 = \frac{4\pi^2}{\Delta^3}\Big[\frac{208}{15}\left(\Delta^9 - D^9\right) + 6\Delta^7 + \frac{282}{5}D^7 - \frac{441}{5}D^5 + \frac{259}{4}D^3 -$$
$$- \frac{119}{8}D + \frac{161}{3D} - \frac{15}{D^3} + \frac{3}{D^5} - \frac{367}{8\Delta}\ln(\Delta + D)\Big] .$$

The components of the tensors quadratic in $\delta\omega$ look like

$$S_1 = \frac{4\pi^2}{\Delta^3}\left[\frac{\Delta^7}{9} + \frac{\Delta^5}{6} - \frac{D^7}{9} + \frac{2}{9}D^5 - \frac{5}{72}D^3 + \frac{D}{48} - \left(\frac{\Delta}{12} + \frac{1}{16\Delta}\right)\ln(\Delta + D)\right] ,$$
$$S_2 = \frac{4\pi^2}{\Delta^3}\left[-\Delta^7 - \frac{\Delta^5}{3} + D^7 - \frac{19}{6}D^5 + \frac{85}{24}D^3 - \frac{27}{16}D + \frac{5}{16\Delta}\ln(\Delta + D)\right] ,$$
$$T_1 = \frac{8\pi^2}{\Delta^3}\left[\frac{\Delta^9}{25} - \frac{D^9}{25} + \frac{9}{50}D^7 - \frac{63}{200}D^5 + \frac{21}{80}D^3 - \frac{D}{80} - \left(\frac{\Delta}{16} + \frac{3}{40\Delta}\right)\ln(\Delta + D)\right] ,$$
$$T_2 = \frac{8\pi^2}{\Delta^3}\left[-\frac{4\Delta^9}{5} + \frac{4D^9}{5} - \frac{18}{5}D^7 + \frac{63}{10}D^5 - \frac{21}{4}D^3 + \frac{15}{8}D - \frac{1}{2D} + \frac{3}{8\Delta}\ln(\Delta + D)\right] .$$

$$Z_1 = \frac{8\pi^2}{\Delta^5}\Big[\frac{29}{90}\left(D^9 - \Delta^9\right) + \frac{53}{504}\Delta^7 - \frac{3919}{2520}D^7 + \frac{523}{180}D^5 - \frac{52723}{20160}D^3 +$$
$$+ \frac{43717}{40320}D - \frac{52}{315D} + \left(\frac{\Delta}{32} + \frac{3}{128\Delta}\right)\ln(\Delta + D)\Big] ,$$
$$Z_2 = \frac{8\pi^2}{\Delta^5}\Big[\frac{17}{45}\left(D^9 - \Delta^9\right) + \frac{11}{504}\Delta^7 - \frac{4339}{2520}D^7 + \frac{2197}{720}D^5 - \frac{53633}{20160}D^3 +$$
$$+ \frac{44137}{40320}D - \frac{52}{315D} + \left(\frac{\Delta^3}{24} + \frac{5}{32}\Delta + \frac{3}{128\Delta}\right)\ln(\Delta + D)\Big] ,$$
$$Z_3 = \frac{8\pi^2}{\Delta^5}\Big[\frac{233}{90}\left(\Delta^9 - D^9\right) - \frac{29}{72}\Delta^7 + \frac{4339}{360}D^7 - \frac{7847}{360}D^5 + \frac{54703}{2880}D^3 -$$
$$- \frac{44077}{5760}D + \frac{52}{45D} - \left(\frac{5}{32}\Delta + \frac{21}{128\Delta}\right)\ln(\Delta + D)\Big] ,$$
$$Z_4 = \frac{8\pi^2}{\Delta^5}\Big[\frac{94}{45}\left(\Delta^9 - D^9\right) - \frac{41}{72}\Delta^7 + \frac{3589}{360}D^7 - \frac{13279}{720}D^5 + \frac{44173}{2880}D^3 -$$
$$- \frac{39217}{5760}D + \frac{52}{45D} - \frac{21}{128\Delta}\ln(\Delta + D)\Big] ,$$
$$Z_5 = \frac{8\pi^2}{\Delta^5}\Big[\frac{114}{5}\left(D^9 - \Delta^9\right) + \frac{33}{8}\Delta^7 - \frac{4269}{40}D^7 + \frac{1519}{80}D^5 - \frac{54213}{320}D^3 +$$
$$+ \frac{44337}{640}D - \frac{57}{5D} + \frac{189}{128\Delta}\ln(\Delta + D)\Big] .$$

$$\Phi_1^{44} = \frac{8\pi^2}{\Delta^3}\left[\frac{1}{25}\left(\Delta^9 - D^9\right) + \frac{\Delta^7}{12} + \frac{\Delta^5}{24} - \frac{29}{300}D^7 - \frac{13}{200}D^5 - \frac{D^3}{480} + \frac{D}{320} + \frac{1}{320\Delta}\ln(\Delta + D)\right] ,$$
$$\Phi_2^{44} = \frac{8\pi^2}{\Delta^3}\left[\frac{4}{5}\left(D^9 - \Delta^9\right) - \frac{3}{4}\Delta^7 - \frac{\Delta^5}{12} - \frac{57}{20}D^7 + \frac{451}{120}D^5 - \frac{209}{96}D^3 + \frac{31}{64}D - \frac{1}{64\Delta}\ln(\Delta + D)\right] ,$$

$$\Phi_1 = \frac{16\pi^2}{\Delta^5}\Big[\frac{11}{150}\left(\Delta^{11} - D^{11}\right) + \frac{\Delta^9}{60} + \frac{\Delta^7}{96} + \frac{D^9}{50} - \frac{43}{2400}D^7 + \frac{7}{4800}D^5 +$$
$$+ \frac{3}{1280}D^3 - \frac{3}{2560}D + \left(\frac{\Delta}{320} + \frac{1}{512\Delta}\right)\ln(\Delta + D)\Big] ,$$

$$
\begin{aligned}
\Phi_2 &= \frac{16\pi^2}{\Delta^5}\left[\frac{11}{15}\left(D^{11}-\Delta^{11}\right)-\frac{3}{40}\Delta^9-\frac{\Delta^7}{96}-\frac{7}{24}D^9+\frac{239}{48}D^7-\frac{49}{120}D^5+\right.\\
&+\left.\frac{119}{768}D^3-\frac{3}{512}D-\left(\frac{\Delta}{128}+\frac{7}{512\Delta}\right)\ln(\Delta+D)\right] ,\\
\Phi_3 &= \frac{16\pi^2}{\Delta^5}\left[\Delta^{11}-D^{11}+\frac{3}{10}\Delta^9-\frac{\Delta^7}{32}+\frac{52}{10}D^9-\frac{1759}{160}D^7+\frac{3829}{320}D^5-\right.\\
&-\left.\frac{1771}{256}D^3-\frac{959}{512}D-\frac{1}{4D}+\frac{63}{512\Delta}\ln(\Delta+D)\right] .
\end{aligned}
$$

The components of kinetic energy tensors which are the cross terms between $\delta\rho$ and $\delta\omega$ are defined as follows

$$
\begin{aligned}
\Psi_1 &= \frac{4\pi^2}{\Delta^3}\left[\frac{16}{9}\Delta^7-\frac{16}{9}D^7+\frac{56}{9}D^5-\frac{70}{9}D^3+\frac{23}{6}D-\frac{2}{D}-\left(\frac{\Delta}{3}-\frac{3}{2\Delta}\right)\ln(\Delta+D)\right] ,\\
\Psi_2 &= \frac{4\pi^2}{\Delta^3}\left[\frac{8}{9}\Delta^7-\frac{8}{9}D^7+\frac{28}{9}D^5-\frac{35}{9}D^3+\frac{D}{6}+\left(\frac{4\Delta}{3}+\frac{3}{2\Delta}\right)\ln(\Delta+D)\right] ,\\
\Psi_3 &= \frac{4\pi^2}{\Delta^3}\left[-8\Delta^7+8D^7-28D^5+35D^3-\frac{31}{2}D+\frac{8}{D}-\frac{15}{2\Delta}\ln(\Delta+D)\right] ,
\end{aligned}
$$

$$
\begin{aligned}
W_1 &= \frac{16\pi^2}{\Delta^3}\left[\frac{2}{25}(D^9-\Delta^9)-\frac{9}{25}D^7+\frac{63}{100}D^5-\frac{21}{40}D^3+\frac{2}{5}D-\left(\frac{\Delta}{8}+\frac{9}{40\Delta}\right)\ln(\Delta+D)\right] ,\\
W_2 &= \frac{16\pi^2}{\Delta^3}\left[\frac{8}{5}(\Delta^9-D^9)+\frac{36}{5}D^7-\frac{63}{5}D^5+\frac{21}{2}D^3-\frac{33}{8}D-\frac{3}{4D}+\frac{1}{4D^3}+\frac{9}{8\Delta}\ln(\Delta+D)\right] ,
\end{aligned}
$$

$$
\begin{aligned}
X_1 &= \frac{8\pi^2}{\Delta^3}\left[\frac{D^7}{18}-\frac{\Delta^7}{18}-\frac{7}{36}D^5+\frac{35}{144}D^3-\frac{D}{96}-\left(\frac{\Delta}{12}+\frac{3}{32\Delta}\right)\ln(\Delta+D)\right] ,\\
X_2 &= \frac{8\pi^2}{\Delta^3}\left[\frac{\Delta^7}{2}-\frac{D^7}{2}+\frac{7}{4}D^5-\frac{35}{16}D^3+\frac{31}{32}D-\frac{1}{2D}+\frac{15}{32\Delta}\ln(\Delta+D)\right].
\end{aligned}
$$

$$
\begin{aligned}
\Theta_1^{44} &= \frac{48\pi^2}{\Delta^3}\left[\frac{1}{150}\left(\Delta^9-D^9\right)+\frac{3}{100}D^7-\frac{21}{400}D^5+\frac{7}{160}D^3-\frac{D}{480}-\left(\frac{\Delta}{96}+\frac{1}{80\Delta}\right)\ln(\Delta+D)\right] ,\\
\Theta_2^{44} &= \frac{48\pi^2}{\Delta^3}\left[\frac{2}{15}\left(D^9-\Delta^9\right)-\frac{3}{5}D^7+\frac{21}{20}D^5-\frac{7}{8}D^3+\frac{5}{16}D-\frac{1}{12D}+\frac{1}{16\Delta}\ln(\Delta+D)\right] ,
\end{aligned}
$$

$$
\begin{aligned}
\Theta_1 &= \frac{96\pi^2}{\Delta^5}\left[\frac{1}{900}\left(\Delta^{11}-D^{11}\right)+\frac{11}{1800}D^9-\frac{11}{800}D^7+\frac{77}{4800}D^5-\frac{D^3}{640}+\right.\\
&+\left.\frac{D}{1280}-\left(\frac{\Delta^3}{192}+\frac{\Delta}{80}+\frac{5}{768\Delta}\right)\ln(\Delta+D)\right] ,\\
\Theta_2 &= \frac{96\pi^2}{\Delta^5}\left[\frac{1}{90}\left(D^{11}-\Delta^{11}\right)-\frac{11}{180}D^9+\frac{11}{80}D^7-\frac{77}{480}D^5+\frac{37}{384}D^3-\right.\\
&-\left.\frac{53}{768}D+\left(\frac{\Delta}{32}+\frac{35}{768\Delta}\right)\ln(\Delta+D)\right] ,\\
\Theta_3 &= \frac{96\pi^2}{\Delta^5}\left[\frac{1}{6}\left(\Delta^{11}-D^{11}\right)+\frac{11}{12}D^9-\frac{33}{16}D^7+\frac{77}{32}D^5-\frac{195}{128}D^3+\right.\\
&+\left.\frac{151}{256}D+\frac{7}{24D}-\frac{1}{24D^3}-\frac{315}{768\Delta}\ln(\Delta+D)\right] .
\end{aligned}
$$

$$
\begin{aligned}
\Xi_1 &= \frac{32\pi^2}{\Delta^5}\left[\frac{29}{180}\left(\Delta^9-D^9\right)+\frac{29}{40}D^7-\frac{203}{160}D^5+\frac{20497}{20160}D^3-\frac{12}{35}D-\right.\\
&- \left.\frac{2}{35D}+\frac{13}{315D^3}+\frac{3}{64}\left(\Delta+\frac{1}{\Delta}\right)\ln(\Delta+D)\right]\ ,\\
\Xi_2 &= \frac{32\pi^2}{\Delta^5}\left[\frac{17}{90}\left(\Delta^9-D^9\right)+\frac{17}{20}D^7-\frac{119}{80}D^5+\frac{1921}{1680}D^3-\frac{2339}{6720}D-\right.\\
&- \left.\frac{2}{35D}+\frac{13}{315D^3}+\left(\frac{\Delta^3}{24}+\frac{3}{32}\Delta+\frac{3}{64\Delta}\right)\ln(\Delta+D)\right]\ ,\\
\Xi_3 &= \frac{32\pi^2}{\Delta^5}\left[\frac{233}{180}\left(D^9-\Delta^9\right)-\frac{233}{40}D^7+\frac{1631}{160}D^5-\frac{23767}{2880}D^3+\frac{449}{160}D+\right.\\
&+ \left.\frac{2}{5D}-\frac{13}{45D^3}-\left(\frac{15}{64}\Delta+\frac{21}{64\Delta}\right)\ln(\Delta+D)\right]\ ,\\
\Xi_4 &= \frac{32\pi^2}{\Delta^5}\left[\frac{47}{45}\left(D^9-\Delta^9\right)-\frac{47}{10}D^7+\frac{329}{40}D^5-\frac{9611}{1440}D^3+\frac{663}{320}D+\right.\\
&+ \left.\frac{13}{20D}-\frac{13}{45D^3}-\frac{21}{64\Delta}\ln(\Delta+D)\right]\ ,\\
\Xi_5 &= \frac{32\pi^2}{\Delta^5}\left[\frac{57}{5}\left(\Delta^9-D^9\right)+\frac{513}{10}D^7-\frac{3591}{40}D^5+\frac{11691}{160}D^3-\frac{7407}{320}D-\right.\\
&- \left.\frac{117}{20D}+\frac{57}{20D^3}+\frac{189}{64\Delta}\ln(\Delta+D)\right]\ .
\end{aligned}
$$

The tensors which are not integrated in the elementary functions read

$$
\begin{aligned}
B_i &= 8\int dy\ \frac{y_i}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ \frac{2\ y^2-\boldsymbol{y}^2}{y^2\ Y^3}\ , & C_{ijk} &= 8\int dy\ \frac{y_i\ y_j\ y_k}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ \frac{1}{y^2\ Y^3}\ ,\\
O_i &= 4\int dy\ \frac{y_i}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ \frac{1}{y^2\ Y^2}\ , & P_{ijk} &= 4\int dy\ \frac{y_i\ y_j\ y_k}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\ \frac{1}{y^2\ Y^2}\ ,\\
Q_{ij} &= 12\int dy \frac{y_i y_j}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\frac{y^2-1}{Y^4}\ , & R_{ijkl} &= 4\int dy \frac{y_i y_j y_k y_l}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\left(\frac{1}{y^2Y^3}+\frac{2}{Y^4}-\frac{4}{y^2Y^4}\right)\ ,\\
U_{ij} &= 2\int dy \frac{y_i y_j}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\frac{1}{y^2Y^2}\ , & V_{ijkl} &= 2\int dy \frac{y_i y_j y_k y_l}{|\boldsymbol{y}+\boldsymbol{\Delta}|^2}\frac{1}{y^2\ Y^2}\ ,
\end{aligned}
$$

where $Y = y^2+1$. The coefficients of the obtained polynomials in Δ and D should be rather peculiarly organized. Because at large distances the corresponding integrals are the decreasing functions. Therefore the coefficients at high powers should be tuned in such a way to cancel each other, up to the order necessary for functions decreasing, at expanding $D=\sqrt{\Delta^2+1}$, when $\Delta\to\infty$. At small Δ the singularity of terms related to $\sim \frac{\ln\Delta}{\Delta}$ what means the sum of all the coefficients, excepting the terms proportional to $\sim \Delta^3\ \ln\Delta$, $\Delta\ \ln\Delta$ and the powers of Δ, should equal to zero.

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